```
> Método Dos Minimos Quadrados
```

Método Dos Minimos Quadrados **(1)**

```
> restart:
> with(plots) :
> with(linalg) :
>
> x1[0] := 1 :
> x2[0] := 2 :
> x3[0] := 3 :
> x4[0] := 4 :
> x5[0] := 5 :
>
> Colocando os pesos :
>
> w1[0] := 1 :;
> w2[0] := 1 :;
> w3[0] := 1 :;
> w4[0] := 1 :;
> w5[0] := 1 :;
>
> Valor da função :
>
> f1[0] := 0.8 :
> f2[0] := 1.2 :
> f3[0] := 0.9 :
> f4[0] := 1.3 :
> f5[0] := 1.5 :
>
> Montando o sitema :
>
> X := array(1..5, [x1[0], x2[0], x3[0], x4[0], x5[0]]) :;
>
  a11 := w1[0] + w2[0] + w3[0] + w4[0] + w5[0] :;
> a12 := w1[0]·x1[0] + w2[0]·x2[0] + w3[0]·x3[0] + w4[0]·x4[0] + w5[0]·x5[0] :;
> a21 := a12 :
> a22 := w1[0]·x1[0]^2 + w2[0]·x2[0]^2 + w3[0]·x3[0]^2 + w4[0]·x4[0]^2 + w5[0]·x5[0]^2 :;
> a13 := a22 :
> a31 := a13 :
> a23 := w1[0]·x1[0]^3 + w2[0]·x2[0]^3 + w3[0]·x3[0]^3 + w4[0]·x4[0]^3 + w5[0]·x5[0]^3 :;
> a33 := w1[0]·x1[0]^4 + w2[0]·x2[0]^4 + w3[0]·x3[0]^4 + w4[0]·x4[0]^4 + w5[0]·x5[0]^4 :;
> a32 := a23 :
>
> b1 := w1[0]·f1[0] + w2[0]·f2[0] + w3[0]·f3[0] + w4[0]·f4[0] + w5[0]·f5[0] :;
> b2 := w1[0]·f1[0]·x1[0] + w2[0]·f2[0]·x2[0] + w3[0]·f3[0]·x3[0] + w4[0]·f4[0]·x4[0]
```

```
        + w5[0]·f5[0]·x5[0] :;
> b3 := w1[0]·f1[0]·x1[0]^2 + w2[0]·f2[0]·x2[0]^2 + w3[0]·f3[0]·x3[0]^2 + w4[0]·f4[0]
      ·x4[0]^2 + w5[0]·f5[0]·x5[0]^2 :;
>
> A1 := Matrix([[a11, a12, a13], [a21, a22, a23], [a31, a32, a33]]);
```

$$A1 := \begin{bmatrix} 5 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{bmatrix} \quad \textbf{(2)}$$

```
> B1 := Vector([b1, b2, b3]);
```

$$B1 := \begin{bmatrix} 5.7 \\ 18.6 \\ 72.0 \end{bmatrix} \quad \textbf{(3)}$$

```
> det(A1);
```

$$700 \quad \textbf{(4)}$$

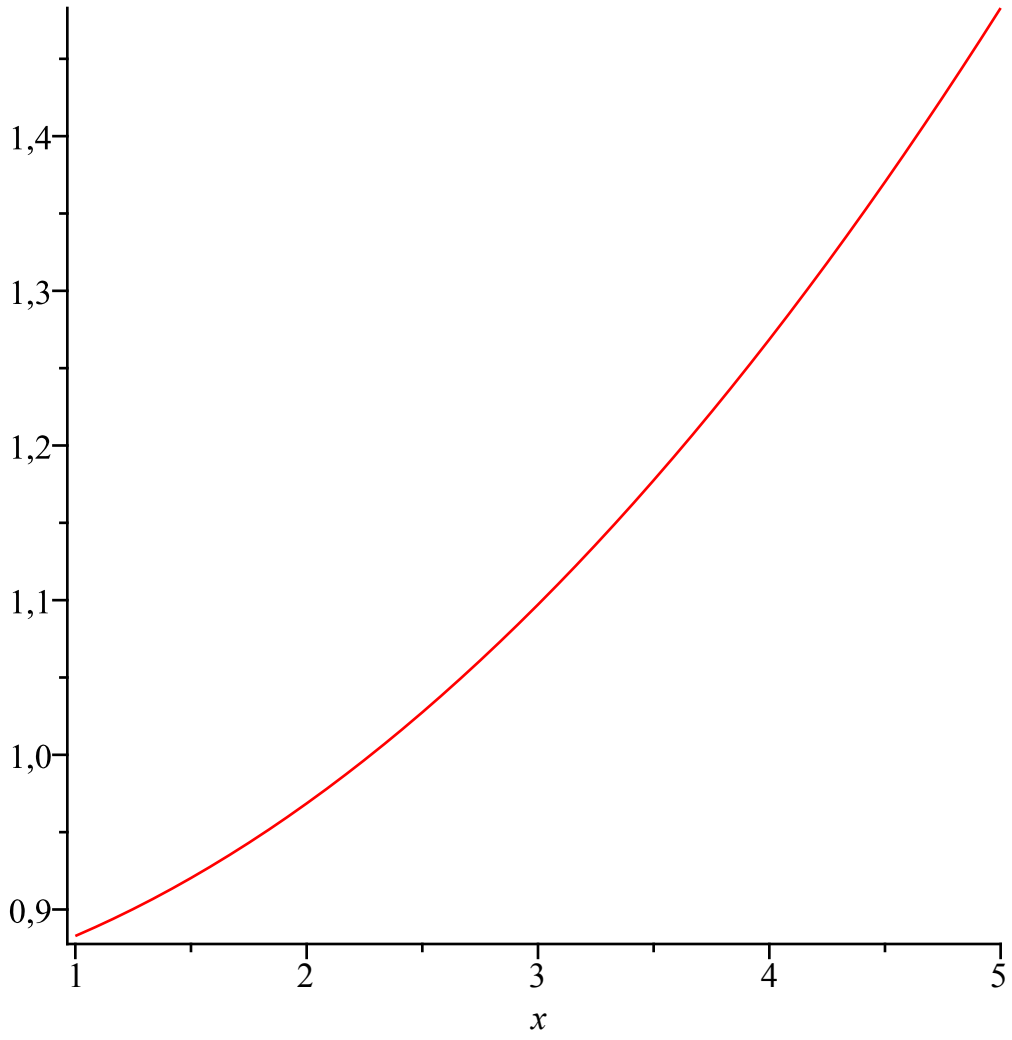
```
> c2 := linsolve(A1, B1);
```

$$c2 := \begin{bmatrix} 0.8400000056 & 0.0214285669 & 0.02142857216 \end{bmatrix} \quad \textbf{(5)}$$

```
> phi(x) := (c2[1] + c2[2]·x + c2[3]·x^2);
```

$$\phi(x) := 0.8400000056 + 0.0214285669\,x + 0.02142857216\,x^2 \quad \textbf{(6)}$$

```
>
>
> plot(phi(x), x = 1 .. 5 );
```

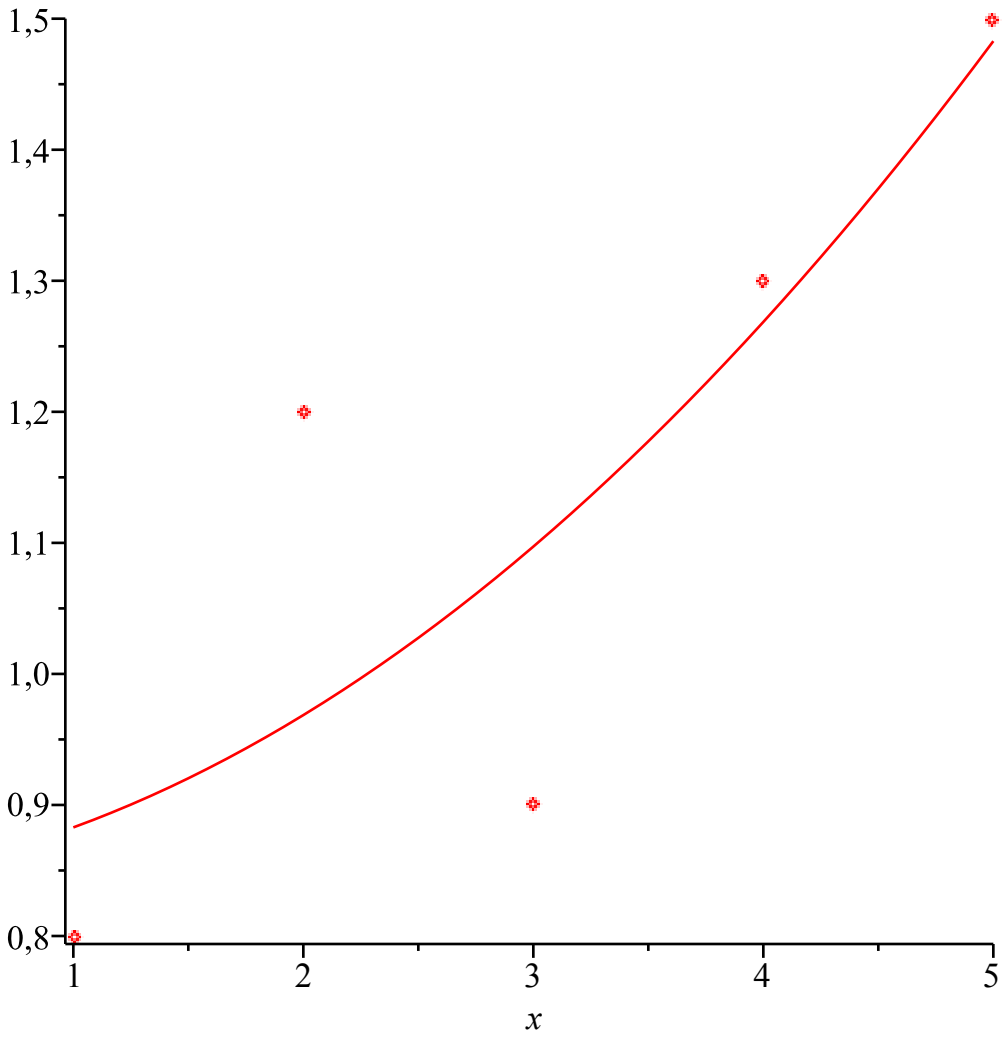



```
> N := 5 :
> F := vector( [ f1[0], f2[0], f3[0], f4[0], f5[0]]) :;
> X := vector( [x1[0], x2[0], x3[0], x4[0], x5[0]]) :;
> pontos := [ [ X[i], F[i]] $i = 1 ..5];
```

$$pontos := [[1, 0.8], [2, 1.2], [3, 0.9], [4, 1.3], [5, 1.5]] \quad \textbf{(7)}$$

```
> plot( pontos ) :;
> gr1 := plot( phi(x), x = 1 .. 5 ) :;
  gr2 := plot( pontos, style = point) :;

> display(gr1, gr2);
```



>

> *Resposta exercício* :

> $x := 6$:

> $\mathrm{phi}(x) := (c2[1] + c2[2]\cdot x + c2[3]\cdot x^2)$;

$$\phi(6) := 1.740000005 \quad \textbf{(8)}$$

>

>

> *Método Dos Minimos Quadrados Polinomios Ortogonais*

Método Dos Minimos Quadrados Polinomios Ortogonais **(9)**

> *restart* :

> *with*(*plots*) :

> *with*(*linalg*) :

>

> $x[1] := 1$:

> $x[2] := 2$:

> $x[3] := 3$:

> $x[4] := 4$:

```
> x[5] := 5 :
>
> Colocando os pesos :
>
> w[1] := 1 :;
> w[2] := 1 :;
> w[3] := 1 :;
> w[4] := 1 :;
> w[5] := 1 :;
>
> Valor da função :
>
> f[1] := 0.8 :
> f[2] := 1.2 :
> f[3] := 0.9 :
> f[4] := 1.3 :
> f[5] := 1.5 :
>
>
> Queremos achar o polinomio p(x) = a + bx;
```

$$\text{Queremos achar o polinomio } p(x) = a + bx \tag{10}$$

```
> Montando o sitema :
> p1(x) := 1 :
> p2(x) := x - α1 :
>
>
> n := 5 :
> k := 1 :
> α1 := 1/n · (sum(w[i]·x[i], i = k..n))
```

$$\alpha 1 := 3 \tag{11}$$

```
> p2(x) := x - α1
```

$$p2(x) := x - 3 \tag{12}$$

```
>
>
> p1(x)
```

$$1 \tag{13}$$

```
> p2(x)
```

$$x - 3 \tag{14}$$

$$\tag{15}$$

```
> Montando o Sistema
```

$$\text{Montando o Sistema} \tag{16}$$

> $a11 := \sum_{i=k}^{n} w[i] \cdot p1(x)$

$$a11 := 5 \quad (17)$$

> $a22 := \sum_{i=k}^{n} (w[i] \cdot x[i] - 3)^2$

$$a22 := 10 \quad (18)$$

>

> $b1 := \sum_{i=k}^{n} w[i] \cdot f[i] \cdot 1$

$$b1 := 5.7 \quad (19)$$

> $b2 := \sum_{i=k}^{n} f[i] \cdot (w[i] \cdot x[i] - 3)$

$$b2 := 1.5 \quad (20)$$

>

> $a := \frac{b1}{a11}$

$$a := 1.140000000 \quad (21)$$

> $b := \frac{b2}{a22}$

$$b := 0.1500000000 \quad (22)$$

>

> $p(x) := a + b \cdot (x - 3)$

$$p(x) := 0.6900000000 + 0.1500000000\, x \quad (23)$$

> $x := 6$

$$x := 6 \quad (24)$$

> $p(x) := a + b \cdot (x - 3)$

$$p(6) := 1.590000000 \quad (25)$$

>

>

> *Método Dos Minimos Quadrados Polinomios Ortogonais*

Método Dos Minimos Quadrados Polinomios Ortogonais (26)

> *restart*:

> *with*(*plots*) :

> *with*(*linalg*) :

>

> $x[1] := 1$:

> $x[2] := 2$:

> $x[3] := 3$:

> $x[4] := 4$:

> $x[5] := 5$:

>

> *Colocando os pesos* :

>

> $w1 := 1$:;
> $w2 := 1$:;
> $w3 := 1$:;
> $w4 := 1$:;
> $w5 := 1$:;

>

> *Valor da função* :

>

> $f[1] := 0.8$:
> $f[2] := 1.2$:
> $f[3] := 0.9$:
> $f[4] := 1.3$:
> $f[5] := 1.5$:

>

> *Queremos achar o polinomio* $p(x) = a + bx + cx^2$:;

>

>

> $n := 5$:
> $k := 1$:

> $\alpha 1 := \frac{1}{n} \cdot \left(\sum_{i=k}^{n} x[i] \right)$

$$\alpha 1 := 3 \quad \textbf{(27)}$$

> $p2(x) := x - \alpha 1$

$$p2(x) := x - 3 \quad \textbf{(28)}$$

>

> $$\alpha 2 := \frac{\sum_{i=k}^{n} \left(x[i] \cdot (x[i] - 3)^2 \right)}{\sum_{i=k}^{n} (x[i] - 3)^2}$$

$$\alpha 2 := 3 \quad \textbf{(29)}$$

> $$\beta := \frac{\sum_{i=k}^{n} \left(x[i] \cdot (x[i] - 3) \right)}{\sum_{i=k}^{n} (1)^2}$$

$$\beta := 2 \quad \textbf{(30)}$$

> *Montando o sitema* :
> $p1(x) := 1$:

> $p2(x) := x - \alpha 1$:

> $p3(x) := x \cdot p2(x) - \alpha 2 \cdot p2(x) - \beta \cdot p1(x)$:

>

> $p1(x)$

$$1 \quad (31)$$

> $p2(x)$

$$x - 3 \quad (32)$$

(33)

> $p3(x)$

$$x\,(x-3) - 3\,x + 7 \quad (34)$$

> *Montando o Sistema*

Montando o Sistema (35)

> $a11 := \sum_{i=k}^{n} p1(x)$

$$a11 := 5 \quad (36)$$

> $a22 := \sum_{i=k}^{n} (x[i]-3)^2$

$$a22 := 10 \quad (37)$$

> $a33 := \sum_{i=k}^{n} (x[i] \cdot (x[i]-3) - 3 \cdot x[i] + 7)^2$

$$a33 := 14 \quad (38)$$

> $b1 := \sum_{i=k}^{n} f[i] \cdot 1$

$$b1 := 5.7 \quad (39)$$

> $b2 := \sum_{i=k}^{n} f[i] \cdot (x[i]-3)$

$$b2 := 1.5 \quad (40)$$

> $b3 := \sum_{i=k}^{n} f[i] \cdot (x[i] \cdot (x[i]-3) - 3 \cdot x[i] + 7)$

$$b3 := 0.3 \quad (41)$$

> $a := \frac{b1}{a11}$

$$a := 1.140000000 \quad (42)$$

> $b := \frac{b2}{a22}$

$$b := 0.1500000000 \quad (43)$$

> $c := \frac{b3}{a33}$

$$c := 0.02142857143 \quad (44)$$

```
> p(x) := a + b·(x − 3) + c·(x (x − 3) − 3 x + 7)
```

$$p(x) := 0.8400000000 + 0.08571428571\,x + 0.02142857143\,x\,(x-3) \quad \textbf{(45)}$$

```
> x := 6
```

$$x := 6 \quad \textbf{(46)}$$

```
> p(x) := a + b·x + c·(x (x − 3) − 3 x + 7)
```

$$p(6) := 2.190000000 \quad \textbf{(47)}$$

```
>
>
>
>
>
>
>
```