```
> restart;
```

RESOLVENDO SISTEMAS NÃO LINEARES PELO MÉTODO DE NEWTON

EXEMPLO 1

Considere o seguinte sistema não linear
x-y^2+2y = 0
2x+y^2 = 6

```
> with(linalg):
> with(plots);
```

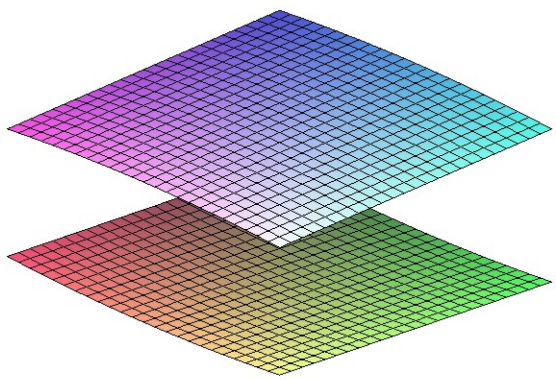
```
> g1:=(x,y)->((x-y*y+2*y));
```

$$g1 := (x, y) \rightarrow x - y^2 + 2\,y$$

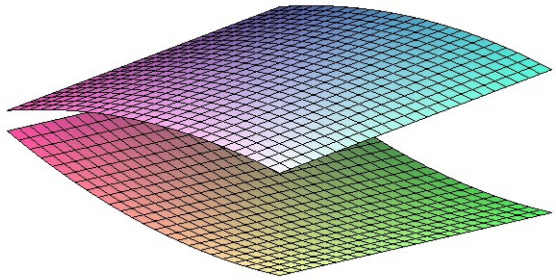
```
> g2:=(x,y)->((2*x+y^2-6));
```

$$g2 := (x, y) \rightarrow 2\,x + y^2 - 6$$

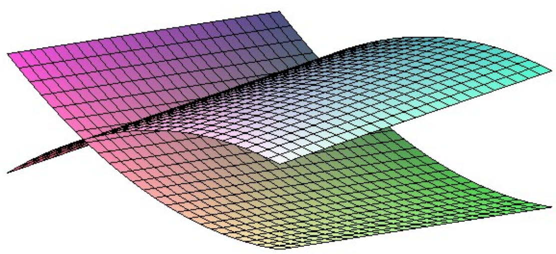
```
> plot3d({x-y*y+2*y,2*x+y^2-6},x=0..1,y=0..1);
```



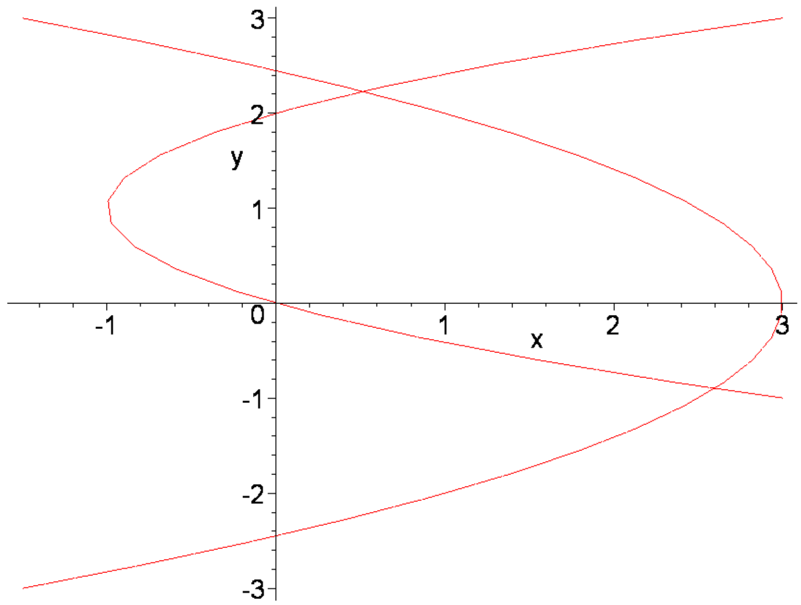
```
> plot3d({x-y*y+2*y,2*x+y^2-6},x=-1..1,y=-1..1);
```



```
> plot3d({x-y^2+2*y,2*x+y^2-6},x=-2..1,y=-2..1);
```



```
> implicitplot({x-y^2+2*y,2*x+y^2-6},x=-3..3,y=-3..3);
```



Note pelo gráfico que temos duas soluções aproximadas no intervalo [-3,3]x[-3,3]. Uma solução X_1=(x_1, y_1), onde x_1 esta em[0,1] e y_1 esta em [2,3]

Uma segunda solução X_2=(x_2, y_2), onde x_2 esta em[2,3] e y_2 esta em [-1,0]. Vamos aplicar o método de Newton para obter as duas soluções.

X^[n+1]=X^n- F[X^n] / J[X^n]..........DENOTANDO   J[X^n]}*Y^n= **-** F[X^n] então   X^[n+1] = X^[n]+Y^[n], para n = 0,1,2......

```
>
> Jaco := (x,y)->array(1..2,1..2,[[1,-2*y+2],[2,2*y]]);
```

$$Jaco := (x, y) \rightarrow \mathrm{array}(1 .. 2, 1 .. 2, [[1, -2\,y + 2], [2, 2\,y]])$$

```
> F:=(x,y)->array(1..2,[[x-y^2+2*y],[2*x+y^2-6]]);
```

$$F := (x, y) \rightarrow \mathrm{array}(1 .. 2, [[x - y^2 + 2\,y], [2\,x + y^2 - 6]])$$

OBTENDO A PRIMEIRA SOLUÇÃO DO SISTEMA NÃO LINEAR. **X=(x_1, y_1)**
TOMAREMOS COMO APROXIMAÇÃO INICIAL O VALOR DE X^0 = [0.5, 2.5]

```
> x_0:=0.5;
```

$$x\_0 := .5$$

```
> y_0:=2.5;
```

$$y\_0 := 2.5$$

```
> X_0:=[x_0, y_0];
```

$$X\_0 := [.5, 2.5]$$

```
> J_0:=evalf(Jaco(x_0,y_0));
```

$$J\_0 := \begin{bmatrix} 1. & -3.0 \\ 2. & 5.0 \end{bmatrix}$$

```
> F_0:=evalf(F(x_0,y_0));
```

$$F\_0 := [[-.75], [1.25]]$$

```
> b:=- F_0;
```

$$b := -F\_0$$

```
> Y_0:=multiply(inverse(J_0),b);
```

$$Y\_0 := \begin{bmatrix} 0 \\ -.2500000000 \end{bmatrix}$$

```
> X_1:=evalf(evalm(X_0+Y_0),5);
```

$$X\_1 := \begin{bmatrix} .5 \\ 2.2500 \end{bmatrix}$$

```
> x_1:=0.5;
```

$$x\_1 := .5$$

```
> y_1:=2.25;
```

$$y\_1 := 2.25$$

```
> X_1:=[x_1, y_1];
```

$$X\_1 := [.5, 2.25]$$

```
> J_1:=evalf(Jaco(x_1,y_1));
```

$$J\_1 := \begin{bmatrix} 1. & -2.50 \\ 2. & 4.50 \end{bmatrix}$$

```
>
> F_1:=evalf(F(x_1,y_1));
```

$$F\_1 := [[-.0625], [.0625]]$$

```
> b:=- F_1;
```

$$b := -F\_1$$

```
>
> Y_1:=multiply(inverse(J_1),b);
```

$$Y\_1 := \begin{bmatrix} .01315789474 \\ -.01973684211 \end{bmatrix}$$

```
> X_2:=evalf(evalm(X_1+Y_1),5);
```

$$X\_2 := \begin{bmatrix} .51316 \\ 2.2303 \end{bmatrix}$$

```
> x_2:=.51316;
> y_2:=2.2303;
```

$$x\_2 := .51316$$

$$y\_2 := 2.2303$$

```
> X_2:=[x_2, y_2];
```

$$X\_2 := [.51316, 2.2303]$$

```
> J_2:=evalf(Jaco(x_2,y_2));
```

$$J\_2 := \begin{bmatrix} 1. & -2.4606 \\ 2. & 4.4606 \end{bmatrix}$$

```
> F_2:=evalf(F(x_2,y_2));
```

$$F\_2 := [[-.00047809], [.00055809]]$$

```
> b:=- F_2;
```

$$b := -F\_2$$

```
> Y_2:=multiply(inverse(J_2),b);
```

$$Y\_2 := \begin{bmatrix} .0000809367072 \\ -.0001614050609 \end{bmatrix}$$

```
> X_3:=evalf(evalm(X_2+Y_2),5);
```

$$X\_3 := \begin{bmatrix} .51324 \\ 2.2301 \end{bmatrix}$$

```
> x_3:=.51324;
```

$$x\_3 := .51324$$

```
> y_3:=2.2301;
```

$$y\_3 := 2.2301$$

```
> X_3:=[x_3, y_3];
```

$$X\_3 := [.51324, 2.2301]$$

Solução aproximada do sistem não linear $X\_3 := [[.51324], [2.2301]]$
com erro menor que 1,6x10^{-4}

SEGUNDA SOLUÇÃO APROXIMADA DO SISTEMA NÃO LINEAR.

considere como solução inicial o seguinte valor X_0=(2.5; -0.5)

```
> x_0:=2.5;
```

$$x\_0 := 2.5$$

```
> y_0:=-0.5;
```

$$y\_0 := -.5$$

```
> X_0:=[x_0, y_0];
```

$$X\_0 := [2.5, -.5]$$

```
> J_0:=evalf(Jaco(x_0,y_0));
```

$$J\_0 := \begin{bmatrix} 1. & 3.0 \\ 2. & -1.0 \end{bmatrix}$$

```
>
> F_0:=evalf(F(x_0,y_0));
```

$$F\_0 := [[1.25], [-.75]]$$

```
> b:=- F_0;
```

$$b := -F\_0$$

```
>
>
> Y_0:=multiply(inverse(J_0),b);
```

$$Y\_0 := \begin{bmatrix} .1428571429 \\ -.4642857143 \end{bmatrix}$$

```
> X_1:=evalf(evalm(X_0+Y_0),5);
```

$$X\_1 := \begin{bmatrix} 2.6429 \\ -.96429 \end{bmatrix}$$

```
> x_1:=2.6429;
```

$$x\_1 := 2.6429$$

```
> y_1:=-.96429;
```

$$y\_1 := -.96429$$

```
> X_1:=[x_1, y_1];
```

$$X\_1 := [2.6429, -.96429]$$

```
> J_1:=evalf(Jaco(x_1,y_1));
```

$$J\_1 := \begin{bmatrix} 1. & 3.92858 \\ 2. & -1.92858 \end{bmatrix}$$

```
> F_1:=evalf(F(x_1,y_1));
```

$$F\_1 := [[-.215535204], [.215655204]]$$

```
> b:=- F_1;
```

$$b := -F\_1$$

```
> Y_1:=multiply(inverse(J_1),b);
```

$$Y\_1 := \begin{bmatrix} -.04409905001 \\ .06608857501 \end{bmatrix}$$

```
> X_2:=evalf(evalm(X_1+Y_1),5);
```

$$X\_2 := \begin{bmatrix} 2.5988 \\ -.89820 \end{bmatrix}$$

```
>
> x_2:=2.5988;
```

$$x\_2 := 2.5988$$

```
> y_2:=-.89820;
```

$$y\_2 := -.89820$$

```
> X_2:=[x_2, y_2];
```

$$X\_2 := [2.5988, -.89820]$$

```
> J_2:=evalf(Jaco(x_2,y_2));
```

$$J\_2 := \begin{bmatrix} 1. & 3.79640 \\ 2. & -1.79640 \end{bmatrix}$$

```
> 
> F_2:=evalf(F(x_2,y_2));
```

$$F\_2 := [[-.004363240], [.004363240]]$$

```
> b:=- F_2;
```

$$b := -F\_2$$

```
> Y_2:=multiply(inverse(J_2),b);
```

$$Y\_2 := \begin{bmatrix} -.0009294167770 \\ .001394125165 \end{bmatrix}$$

```
> X_3:=evalf(evalm(X_2+Y_2),5);
```

$$X\_3 := \begin{bmatrix} 2.5979 \\ -.89681 \end{bmatrix}$$

```
> x_3:=2.5979;
```

$$x\_3 := 2.5979$$

```
> y_3:=-.89681;
```

$$y\_3 := -.89681$$

```
> X_3:=[x_3, y_3];
```

$$X\_3 := [2.5979, -.89681]$$

Solução aproximada do sistem não linear X_3= [[2.5979], [−.89681]]
com erro menor que 1,4x10^{-3}

```
> 
```

EXEMPLO 2

SISTEMA NÃO LINEAR
x*sin(x)*cos(y)=0
x*cos(x)*cos(y)=0
x*sin(y)+z =0
APROXIMAÇÃO INICIAL X^0=(x_0,y_0, z_0) = (-Pi/2, Pi ,0)

```
> Jaco := (x,y,z)->array(1..3,1..3,[[(sin(x)+cos(x)*x)*cos(y),-x*sin(x)*sin(y),0],[(cos(x)-x*sin(x))*cos(y),-(x*cos(x))*sin(y),0],[sin(y),x*cos(y),1]]);
> 
```

$$Jaco := (x, y, z) \rightarrow \mathrm{array}(1\,..\,3, 1\,..\,3, [[(\sin(x)+\cos(x)\,x)\cos(y), -x\sin(x)\sin(y), 0], [(\cos(x)-x\sin(x))\cos(y), -x\cos(x)\sin(y), 0], [\sin(y), x\cos(y), 1]])$$

```
> x_0:=-Pi/2;
```

$$x\_0 := -\frac{1}{2}\pi$$

```
> y_0:=-Pi;
```

$$y\_0 := -\pi$$

```
> z_0:=0;
```

$$z\_0 := 0$$

```
> 
> f1:=(x,y,z)->(x*sin(x)*cos(y));
```

$$f1 := (x, y, z) \rightarrow x\sin(x)\cos(y)$$

```
> f2:=(x,y,z)->(x*cos(x)*cos(y));
```

$$f2 := (x, y, z) \rightarrow x\cos(x)\cos(y)$$

```
> f3:=(x,y,z)->(x*sin(y)+z);
```

$$f3 := (x, y, z) \rightarrow x\sin(y)+z$$

```
> 
> F:=(x,y,z)->array(1..3,[[x*sin(x)*cos(y)],[x*cos(x)*cos(y)],[x*sin(y)+z]]);
```

$$F := (x, y, z) \rightarrow \mathrm{array}(1\,..\,3, [[x\sin(x)\cos(y)], [x\cos(x)\cos(y)], [x\sin(y)+z]])$$

```
> X_0:=[x_0], [y_0],[z_0]];
```

$$X\_0 := \left[\left[-\frac{1}{2}\pi\right], [-\pi], [0]\right]$$

```
> J_0:=evalf(Jaco(x_0,y_0,z_0));
```

$$J\_0 := \begin{bmatrix} 1. & 0 & 0 \\ 1.570796327 & 0 & 0 \\ 0 & 1.570796327 & 1. \end{bmatrix}$$

```
> c:=det(J_0);
```

$$c := 0$$

```
> F_0:=evalf(F(x_0,y_0,z_0));
```

$$F\_0 := [[-1.570796327], [0], [0]]$$

```
> b:=-F_0;
```

$$b := -F\_0$$

```
> Y_0:=evalm(multiply(inverse(J_0), b));
>
Error, (in inverse) singular matrix
```

MATRIZ JACOBIANA É SINGULAR

```
>
```

EXEMPLO 3

SISTEMA NÃO LINEAR
x*sin(x)*cos(y)-z =0
x*cos(x)*cos(y)=0
x*sin(y)+z =0
APROXIMAÇÃO INICIAL X^0=(x_0,y_0, z_0) = (-Pi/2, Pi ,0)

```
> Jaco :=
  (x,y,z)->array(1..3,1..3,[[(sin(x)+cos(x)*x)*cos(y),-x*sin(x)*sin(y),-1],[(cos(x)-x*sin(x
  ))*cos(y),-(x*cos(x))*sin(y),0],[sin(y),x*cos(y),1]]);
>
```

$$Jaco := (x, y, z) \to \text{array}(1..3, 1..3, [[(\sin(x)+\cos(x)\,x)\cos(y), -x\sin(x)\sin(y), -1], [(\cos(x)-x\sin(x))\cos(y), -x\cos(x)\sin(y), 0], [\sin(y), x\cos(y), 1]])$$

```
> x_0:=-Pi/2;
```

$$x\_0 := -\frac{1}{2}\pi$$

```
> y_0:=-Pi;
```

$$y\_0 := -\pi$$

```
> z_0:=0;
```

$$z\_0 := 0$$

```
>
> f1:=(x,y,z)->(x*sin(x)*cos(y)-z);
```

$$f1 := (x, y, z) \to x\sin(x)\cos(y) - z$$

```
> f2:=(x,y,z)->(x*cos(x)*cos(y));
```

$$f2 := (x, y, z) \to x\cos(x)\cos(y)$$

```
> f3:=(x,y,z)->(x*sin(y)+z);
```

$$f3 := (x, y, z) \to x\sin(y) + z$$

```
>
> F:=(x,y,z)->array(1..3,[[x*sin(x)*cos(y)-z],[x*cos(x)*cos(y)],[x*sin(y)+z]]);
```

$$F := (x, y, z) \to \text{array}(1..3, [[x\sin(x)\cos(y) - z], [x\cos(x)\cos(y)], [x\sin(y) + z]])$$

```
> X_0:=[[x_0], [y_0],[z_0]];
```

$$X\_0 := \left[\left[-\frac{1}{2}\pi\right], [-\pi], [0]\right]$$

```
> J_0:=evalf(Jaco(x_0,y_0,z_0));
```

$$J\_0 := \begin{bmatrix} 1. & 0 & -1. \\ 1.570796327 & 0 & 0 \\ 0 & 1.570796327 & 1. \end{bmatrix}$$

```
> c:=det(J_0);
```

$$c := -2.467401101$$

```
> F_0:=evalf(F(x_0,y_0,z_0));
```

$$F\_0 := [[-1.570796327], [0], [0]]$$

```
> b:=-F_0;
```

$$b := -F\_0$$

```
>
> Y_0:=evalm(multiply(inverse(J_0), b));
```

$$Y\_0 := \begin{bmatrix} 0 \\ 1.000000000 \\ -1.570796327 \end{bmatrix}$$

```
> X_1:=evalf(evalm(X_0+Y_0),5);
```

$$X\_1 := \begin{bmatrix} -1.5708 \\ -2.1416 \\ -1.5708 \end{bmatrix}$$

```
> x_1:=-1.5708;
>
```

$$x\_1 := -1.5708$$

```
> y_1:=-2.1416;
```

$$y\_1 := -2.1416$$

```
> z_1:=-1.5708;
```

$$z\_1 := -1.5708$$

```
>
> J_1:=evalf(Jaco(x_1,y_1,z_1));
```

$$J\_1 := \begin{bmatrix} .5403053701 & 1.321776388 & -1. \\ .8487185569 & .4855155773\ 10^{-5} & 0 \\ -.8414670155 & .8487165723 & 1. \end{bmatrix}$$

```
>
> F_1:=evalf(F(x_1,y_1,z_1));
```

$$F\_1 := [[.7220834277], [-.3117510045\ 10^{-5}], [-.249023612]]$$

```
> b:=- F_1;
```

$$b := -F\_1$$

```
>
>
> Y_1:=evalm(multiply(inverse(J_1), b));
```

$$Y\_1 := \begin{bmatrix} .4919993625\ 10^{-5} \\ -.2179497205 \\ .4340052918 \end{bmatrix}$$

```
>
> X_2:=evalf(evalm(X_1+Y_1),5);
```

$$X\_2 := \begin{bmatrix} 2.6429 \\ -1.1822 \\ -1.1368 \end{bmatrix}$$

```
> x_2:=2.6429;
```

$$x\_2 := 2.6429$$

```
> y_2:=-1.1822;
```

$$y\_2 := -1.1822$$

```
> z_2:=-1.1368;
```

$$z\_2 := -1.1368$$

```
> J_2:=evalf(Jaco(x_2,y_2,z_2));
```

$$J\_2 := \begin{bmatrix} -.6981952038 & 1.169795892 & -1. \\ -.8116762518 & -2.147966610 & 0 \\ -.9254418088 & 1.001367776 & 1. \end{bmatrix}$$

```
>
> F_2:=evalf(F(x_2,y_2,z_2));
```

$$F\_2 := [[1.615732001], [-.8794097794], [-3.582650156]]$$

```
> b:=- F_2;
```

$$b := -F\_2$$

```
>
>
> Y_2:=evalm(multiply(inverse(J_2), b));
```

$$Y\_2 := \begin{bmatrix} -1.168466797 \\ .0321266498 \\ 2.469131537 \end{bmatrix}$$

```
>
>
> X_3:=evalf(evalm(X_2+Y_2),5);
```

$$X\_3 := \begin{bmatrix} 1.4744 \\ -1.1501 \\ 1.3323 \end{bmatrix}$$

```
> x_3:=1.4744;
```

$$x\_3 := 1.4744$$

```
> y_3:=-1.1501;
```

$$y\_3 := -1.1501$$

```
> z_3:=1.3323;
```

$$z\_3 := 1.3323$$

```
> J_3:=evalf(Jaco(x_3,y_3,z_3));
```

$$J\_3 := \begin{bmatrix} .4644543365 & 1.339591276 & -1. \\ -.5600369038 & .1295331454 & 0 \\ -.9128047844 & .6021393018 & 1. \end{bmatrix}$$

```
>
> F_3:=evalf(F(x_3,y_3,z_3));
```

$$F\_3 := [[-.7329561474], [.05795416542], [-.013539374]]$$

```
> b:=- F_3;
```

$$b := -F\_3$$

```
>
>
> Y_3:=evalm(multiply(inverse(J_3), b));
```

$$Y\_3 := \begin{bmatrix} .2032586908 \\ .4313814985 \\ -.06067687489 \end{bmatrix}$$

```
>
>
>
> X_4:=evalf(evalm(X_3+Y_3),5);
```

$$X\_4 := \begin{bmatrix} 1.6777 \\ -.71872 \\ 1.2716 \end{bmatrix}$$

```
> x_4:=1.6777;
```

$$x\_4 := 1.6777$$

```
> y_4:=-.71872;
```

$$y\_4 := -.71872$$

```
> z_4:=1.2716;
```

$$z\_4 := 1.2716$$

```
> J_4:=evalf(Jaco(x_4,y_4,z_4));
```

$$J\_4 := \begin{bmatrix} .6136200595 & 1.098328199 & -1. \\ -1.335818666 & -.1178646626 & 0 \\ -.6584218207 & 1.262719438 & 1. \end{bmatrix}$$

```
>
> F_4:=evalf(F(x_4,y_4,z_4));
```

$$F\_4 := [[-.016089122], [-.1347323744], [.166965711]]$$

```
> b:=- F_4;
```

$$b := -F\_4$$

```
>
>
> Y_4:=evalm(multiply(inverse(J_4), b));
```

$$Y\_4 := \begin{bmatrix} -.09506374726 \\ -.06570626101 \\ -.1465891836 \end{bmatrix}$$

```
>
>
> X_5:=evalf(evalm(X_4+Y_4),5);
```

$$X\_5 := \begin{bmatrix} 1.5826 \\ -.78443 \\ 1.1250 \end{bmatrix}$$

```
> x_5:=1.5826;
```

$$x\_5 := 1.5826$$

```
> y_5:=-.78443;
```

$$y\_5 := -.78443$$

```
> z_5:=1.1250;
```

$$z\_5 := 1.1250$$

```
> J_5:=evalf(Jaco(x_5,y_5,z_5));
```

$$J\_5 := \begin{bmatrix} .6945201594 & 1.117905346 & -1. \\ -1.128426415 & -.01319600223 & 0 \\ -.7064218550 & 1.120150107 & 1. \end{bmatrix}$$

```
>
> F_5:=evalf(F(x_5,y_5,z_5));
```

$$F\_5 := [[-.004927925], [-.01322157878], [.007016772]]$$

```
> b:=- F_5;
```

$$b := -F\_5$$

```
>
>
> Y_5:=evalm(multiply(inverse(J_5), b));
```

$$Y\_5 := \begin{bmatrix} -.01170518605 \\ -.000995577906 \\ -.01417037455 \end{bmatrix}$$

```
>
>
> X_6:=evalf(evalm(X_5+Y_5),5);
```

$$X\_6 := \begin{bmatrix} 1.5709 \\ -.78543 \\ 1.1108 \end{bmatrix}$$

```
> x_6:=1.5709;
```

$$x\_6 := 1.5709$$

```
> y_6:=-.78543;
```

$$y\_6 := -.78543$$

```
> z_6:=1.1108;
```

$$z\_6 := 1.1108$$

```
> J_6:=evalf(Jaco(x_6,y_6,z_6));
```

$$J\_6 := \begin{bmatrix} .7069691093 & 1.110829400 & -1. \\ -1.110831978 & -.0001151632446 & 0 \\ -.7071292927 & 1.110758678 & 1. \end{bmatrix}$$

```
>
> F_6:=evalf(F(x_6,y_6,z_6));
```

$$F\_6 := [[-.000041327], [-.0001151559121], [-.000029406]]$$

```
> b:=- F_6;
```

$$b := -F\_6$$

```
>
>
> Y_6:=evalm(multiply(inverse(J_6), b));
```

$$Y\_6 := \begin{bmatrix} -.0001036696641 \\ .00003183146081 \\ -.00007925892750 \end{bmatrix}$$

```
>
>
>
> X_7:=evalf(evalm(X_6+Y_6),5);
```

$$X\_7 := \begin{bmatrix} 1.5708 \\ -.78540 \\ 1.1107 \end{bmatrix}$$

Após iterações obtemos como solução aproximada o vetor X=(1.5708, -0.78540, 1.1107)
O erro nesse caso 1x10^{-4}