

RAÍZES COMPLEXAS

Determine a raiz da função $f(z) = z^2 + 1$. Como $z = x + iy$ então $z^2 = x^2 - y^2 - i2xy$. Assim $f(z) = z^2 + 1 = U(x, y) + iV(x, y)$, onde $U(x, y) = x^2 - y^2 + 1$ e $V(x, y) = 2xy$.

MÉTODO DE NEWTON-RAPHSON

```
[ > restart ;
[ > with(linalg) :
Warning, new definition for norm
Warning, new definition for trace
[ > with(plots) :
[ > U:=x^2-y^2+1;
                                     U := x2 - y2 + 1
[ > V:=2*x*y;
                                     V := 2 x y
[ > DUx:=diff(U, x) ;
                                     DUx := 2 x
[ > DVx:=diff(V, x) ;
                                     DVx := 2 y
[ > x0:=1.0;
                                     x0 := 1.0
[ > y0:=1.0;
                                     y0 := 1.0
[ > U0:=subs(x=x0, y=y0, U) ;
[ > V0:=subs(x=x0, y=y0, V) ;
                                     U0 := .0053878320
                                     V0 := .003422121291
[ > DU0:=subs(x=x0, y=y0, DUx) ;
                                     DU0 := .00343137254
[ > DV0:=subs(x=x0, y=y0, DVx) ;
                                     DV0 := 1.994607843
[ > Deltax:= evalf (U0*DU0+V0*DVO) / (DU0^2+DV0^2) ;
                                     Deltax := .001720328117
[ > Deltay:=evalf((V0*DU0-U0*DVO) / (DU0^2+DV0^2)) ;
                                     Deltay := -.002698239121
[ > ERRO:=max(abs(Deltax), abs(Deltay)) ;
                                     ERRO := .002698239121
[ > x1:=x0-Deltax;
                                     x1 := -.4641847 10-5
[ > y1:=y0-Deltay;
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[                                     y1 := 1.000002161
[ > x0:=x1; y0:=y1;
[                                     x0 := -.4641847 10-5
[                                     y0 := 1.000002161
[ x=(x^1, x^2, x^3, x^4)=(0.25; -0.075 ;0.0017156;-0.000000464)^t ....note que esta convergindo para
[ zero
[ y=(y^1, y^2, y^3, y^4)=(0.75; 0.975; 0.997304; 1.000000021)^t.... note que esta convergindo para 1
[ Nesse caso o erro é 0.0027, com quatro iterações. A solução z=x+iy= 0 + i*1
[ >

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EXEMPLO 2

$F(z)=\exp(z)-z^2$. Como $z=x+yi$ então $\exp(z)=\exp(x).\exp(yi)=\exp(x)*(\cos(y)+i \sin(y))$.

Logo $F(z)=U(x,y)+iV(x,y)$, onde

$U(x,y)=\exp(x)*\cos(y)-x^2+y^2$

$V(x,y)=\exp(x)*\sin(y)-2xy$

```

[ > U:=exp(x)*cos(y)-x^2+y^2;
[                                     U := ex cos(y) - x2 + y2
[ > V:=exp(x)*sin(y)-2*x*y;
[                                     V := ex sin(y) - 2 x y
[ > DUx:=diff(U, x);
[                                     DUx := ex cos(y) - 2 x
[ > DVx:=diff(V, x);
[                                     DVx := ex sin(y) - 2 y
[ > x0:=1.6;
[                                     x0 := 1.6
[ > y0:=1.5;
[                                     y0 := 1.5
[ > U0:=evalf(subs(x=x0, y=y0, U));
[ > V0:=evalf(subs(x=x0, y=y0, V));
[                                     U0 := .0403636535
[                                     V0 := .140625011
[ > DU0:=evalf(subs(x=x0, y=y0, DUx));
[                                     DU0 := -2.849636347
[ > DV0:=evalf(subs(x=x0, y=y0, DVx));
[                                     DV0 := 1.940625011
[ > Deltax:= evalf((U0*DU0+V0*DV0)/(DU0^2+DV0^2));
[                                     Deltax := .01328223675
[ > Deltay:=evalf((V0*DU0-U0*DV0)/(DU0^2+DV0^2));
[                                     Deltay := -.04030309703

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> ERRO:=max(abs(Deltax), abs(Deltay));
                                ERRO := .04030309703
> x1:=x0-Deltax;
                                x1 := 1.586717763
> y1:=y0-Deltay;
                                y1 := 1.540303097
> x0:=x1; y0:=y1;
                                x0 := 1.586717763
                                y0 := 1.540303097

x=(x^1, x^2, x^3, x^4)=(1.6; 1.5867 ;1.588048191; 1.588047265)^t ....note que esta convergindo para
1.58805
y=(y^1, y^2, y^3, y^4)=(1.5; 1.5403; 1.540222566; 1.540223501)^t.... note que esta convergindo para
1.540223
O erro é .9353306571 10-6
>

UM ALGORITMO MELHOR
> U:=exp(x)*cos(y)-x^2+y^2;
                                U := ex cos(y) - x2 + y2
> V:=exp(x)*sin(y)-2*x*y;
                                V := ex sin(y) - 2 x y
> DUx:=diff(U, x);
                                DUx := ex cos(y) - 2 x
> DVx:=diff(V, x);
                                DVx := ex sin(y) - 2 y

> x[1]:=1.6:
> y[1]:=1.5:
>
> for i from 1 to 3 do
> U0:=evalf(subs(x=x[i], y=y[i], U));
> V0:=evalf(subs(x=x[i], y=y[i], V));
> DU0:=evalf(subs(x=x[i], y=y[i], DUx));
> DV0:=evalf(subs(x=x[i], y=y[i], DVx));
> Deltax:= evalf((U0*DU0+V0*DV0) / (DU0^2+DV0^2));
> Deltay:=evalf((V0*DU0-U0*DV0) / (DU0^2+DV0^2));
> ERRO[i]:=max(abs(Deltax), abs(Deltay));
> x[i+1]:=x[i]-Deltax;

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> y[i+1]:=y[i]-Deltay;  
>  
>  
> od;
```

```
U0 := .0403636535  
V0 := .140625011  
DU0 := -2.849636347  
DV0 := 1.940625011  
Deltax := .01328223675  
Deltay := -.04030309703  
ERRO1 := .04030309703  
x2 := 1.586717763  
y2 := 1.540303097  
U0 := .0038784266  
V0 := -.002644718  
DU0 := -3.024417471  
DV0 := 1.804801657  
Deltax := -.001330427849  
Deltay := .00008053108259  
ERRO2 := .001330427849  
x3 := 1.588048191  
y3 := 1.540222566  
U0 := -.11096 10-5  
V0 := .4509 10-5  
DU0 := -3.026485988  
DV0 := 1.811454696  
Deltax := .9264569944 10-6  
Deltay := -.9353306571 10-6  
ERRO3 := .9353306571 10-6  
x4 := 1.588047265  
y4 := 1.540223501
```

NOTE QUE O ERRO, APÓS 3 ITERAÇÕES, É APROXIMADAMENTE

```
Deltax := -.9353306571 10-6
```

```
>
```

```

[ >
[ > U:=x^2-y^2+1;
                                     U := x2 - y2 + 1
[ > V:=2*x*y;
                                     V := 2 x y
[ > DUx:=diff(U, x);
                                     DUx := 2 x
[ > DVx:=diff(V, x);
                                     DVx := 2 y
[ >
[ > x[1]:=1.0:
[ > y[1]:=1.0:
[ >
[ >
[ >
[ > for i from 1 to 5 do
[ > U0:=evalf(subs(x=x[i],y=y[i], U));
[ > V0:=evalf(subs(x=x[i],y=y[i], V));
[ > DU0:=evalf(subs(x=x[i],y=y[i], DUx));
[ > DV0:=evalf(subs(x=x[i],y=y[i], DVx));
[ > Deltax:= evalf(U0*DU0+V0*DV0)/(DU0^2+DV0^2);
[ > Deltay:=evalf((V0*DU0-U0*DV0)/(DU0^2+DV0^2));
[ > ERRO[i]:=max(abs(Deltax), abs(Deltay));
[ > x[i+1]:=x[i]-Deltax;
[ > y[i+1]:=y[i]-Deltay;
[ >
[ >
[ >
[ > od;
[ >
[ >
[ >
                                     U0 := 1.00
                                     V0 := 2.00
                                     DU0 := 2.0
                                     DV0 := 2.0
                                     Deltax := .7500000000
                                     Deltay := .2500000000
                                     ERRO1 := .7500000000
                                     x2 := .2500000000
                                     y2 := .7500000000

```

$U0 := .5000000000$
 $V0 := .3750000000$
 $DU0 := .5000000000$
 $DV0 := 1.500000000$
 $Deltax := .3250000000$
 $Deltay := -.2250000000$
 $ERRO_2 := .3250000000$
 $x_3 := -.0750000000$
 $y_3 := .9750000000$
 $U0 := .0550000000$
 $V0 := -.1462500000$
 $DU0 := -.1500000000$
 $DV0 := 1.950000000$
 $Deltax := -.07671568627$
 $Deltay := -.02230392157$
 $ERRO_3 := .07671568627$
 $x_4 := .00171568627$
 $y_4 := .9973039216$
 $U0 := .0053878320$
 $V0 := .003422121291$
 $DU0 := .00343137254$
 $DV0 := 1.994607843$
 $Deltax := .001720328117$
 $Deltay := -.002698239121$
 $ERRO_4 := .002698239121$
 $x_5 := -.4641847 \cdot 10^{-5}$
 $y_5 := 1.000002161$
 $U0 := -.4322 \cdot 10^{-5}$
 $V0 := -.9283714062 \cdot 10^{-5}$
 $DU0 := -.9283694 \cdot 10^{-5}$
 $DV0 := 2.000004322$
 $Deltax := -.4641836970 \cdot 10^{-5}$
 $Deltay := .2161016877 \cdot 10^{-5}$
 $ERRO_5 := .4641836970 \cdot 10^{-5}$

$$x_6 := -.10030 \cdot 10^{-10}$$

$$y_6 := 1.000000000$$

Após 5 iterações para o exemplo começando com os dados iniciais (1.0, 1.0) obtemos o erro Erro=
.4641836970 10^{-5}