METODO DOS MÍNIMOS QUADRADOS

```
> restart:
> with(plots):
> with(linalg):
Warning, new definition for norm
Warning, new definition for trace
```

POLINÔMIOS ORTOGONAIS

Ajuste de curvas por polinomios de grau 1 e 2.

Recorrência: P_{i+1}=(x-alpha{i+1}P_{i}+beta_i P_{i-1}

POLINÔMIOS DE GRAU 1

```
> x[1]:=0.0:;
> x[2]:=0.25:
> x[3]:=0.5:
> x[4]:=0.75:
> x[5]:=1.0:
> f[1]:=1.0:;
> f[2]:=2.0:
> f[3]:=1.0:
> f[4]:=0.0:
> f[5]:=1.0:
> N:=5:
> P_0:= x->1;
```

$$P\_0 := 1$$

```
> ALPHA1:=0;
```

$$ALPHA1 := 0$$

```
> ALPHA1:=ALPHA1+(1/N)*(x[1]+x[2]+x[3]+x[4]+x[5]);
```

$$ALPHA1 := .5000000000$$

```
> P_1:=x->x-ALPHA1;
```

$$P\_1 := x \rightarrow x - ALPHA1$$

```
> a11:=0:
> a22:=0:
> b1:=0:
> b2:=0:
> for i from 1 to N do
> a11:=a11 +1;
> a22:=a22+(P_1(x[i]))^2;
> b1:=b1+f[i]:
> b2:=b2+f[i]*P_1(x[i]):
> od:;
```

```
> a_0:=b1/a11;
```

$a\_0 := 1.000000000$

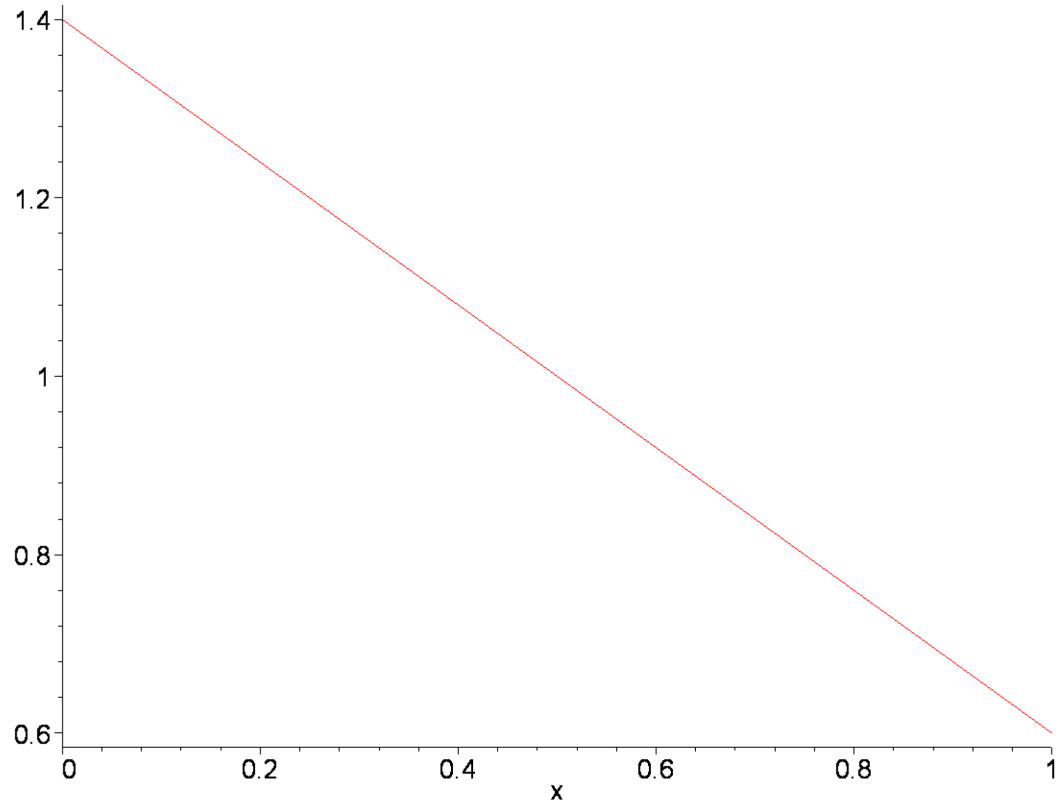
```
>
> a_1:=b2/a22;
```

$a\_1 := -.8000000000$

```
> Q1(x):=a_0+a_1*P_1(x);
```

$Q1(x) := 1.400000000 - .8000000000\ x$

```
> z4:=plot([Q1(x)], x=0..1.0, color=[blue], style=[line]):
> plot([Q1(x)], x=0.0..1.0, color=[red], style=[line]);
```




```
> L:= vector([f[1],f[2], f[3], f[4],f[5]]);
```

$L := [1.0, 2.0, 1.0, 0, 1.0]$

```
> L := vector([1.0, 2.0, 1.0, 0, 1.0]);
```

$L := [1.0, 2.0, 1.0, 0, 1.0]$

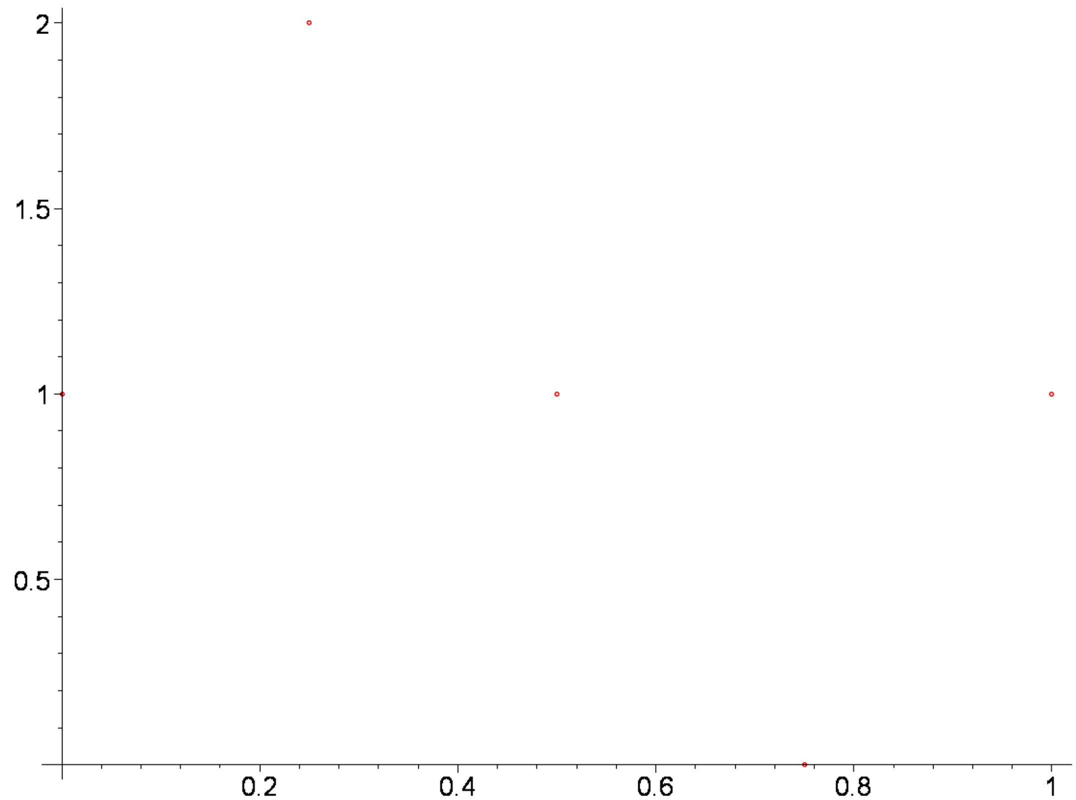
```
> X:=vector([x[1],x[2],x[3],x[4],x[5]]);
```

$X := [0, .25, .5, .75, 1.0]$

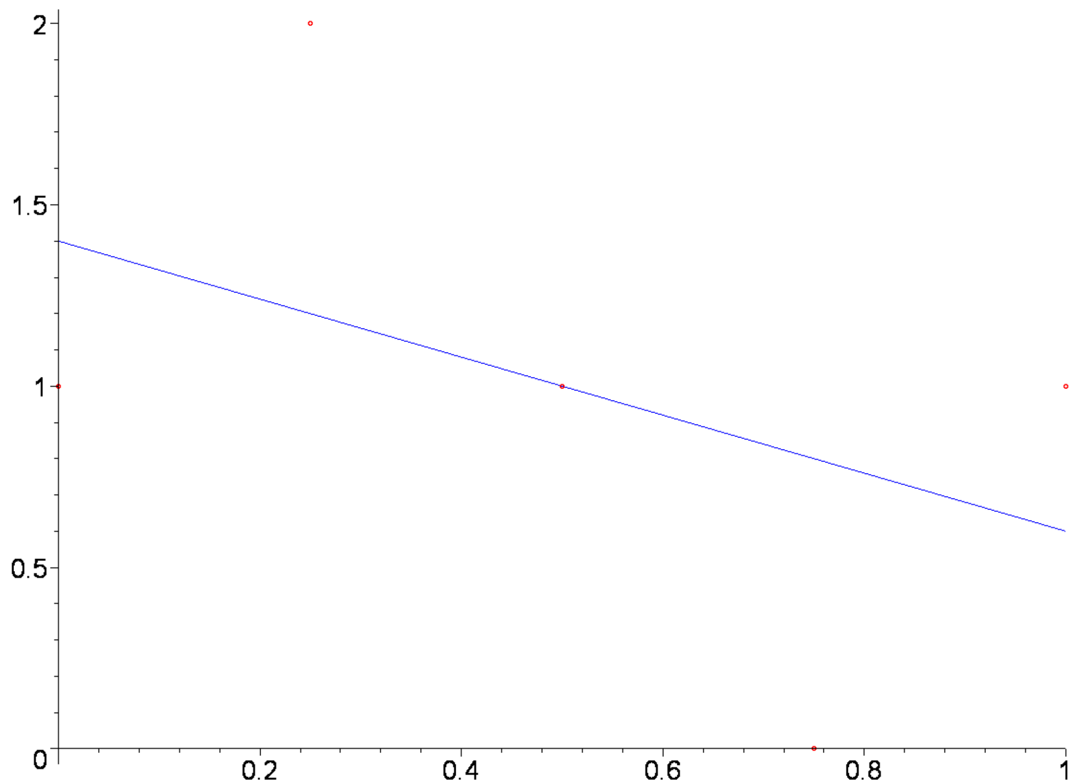
```
> ZZ1:= [[ X[j], L[j]] $j=1..5];
```

$ZZ1 := [[0, 1.0], [.25, 2.0], [.5, 1.0], [.75, 0], [1.0, 1.0]]$

```
> plot(ZZ1,style=point,symbol=circle);
```




```
> display(plot([ZZ1],style=[point],symbol=circle),z4);
```



POLINÔMIO DE GRAU 2

USANDO POLINÔMIOS ORTOGONAIS

Recorrência: P_{i+1}=(x-alpha{i+1}P_{i}+beta_i P_{i-1}

```
> BETA0:=0:;
> gama0:=0:;
> gama1:=0:;
> Q1 := x ->a_0+a_1*P_1(x);
```

$$Q1 := x \rightarrow a\_0 + a\_1 \; \mathrm{P\_1}(x)$$

```
> for i from 1 to N do
> BETA0:=BETA0+x[i]*(P_1(x[i]));
> gama0:=gama0+(x[i]*(P_1(x[i])^2));
> gama1:=gama1+(P_1(x[i])^2):
> od:;
> BETA1:=(1/N)*BETA0;
```

$$BETA1 := .1250000000$$

```
> ALPHA2:=gama0/gama1;
```

$$ALPHA2 := .5000000000$$

```
> P_2:=x ->(x-ALPHA2)*P_1(x)-BETA1*P_0(x);
```

$$P\_2 := x \rightarrow (x - ALPHA2)\; \mathrm{P\_1}(x) - BETA1 \; \mathrm{P\_0}(x)$$

```
> a33:=0:
> b3:=0:
> for i from 1 to N do
> a33:=a33+(P_2(x[i]))^2:
> b3:=b3+f[i]*P_2(x[i]):
> od:;
> a_2:=b3/a33;
```

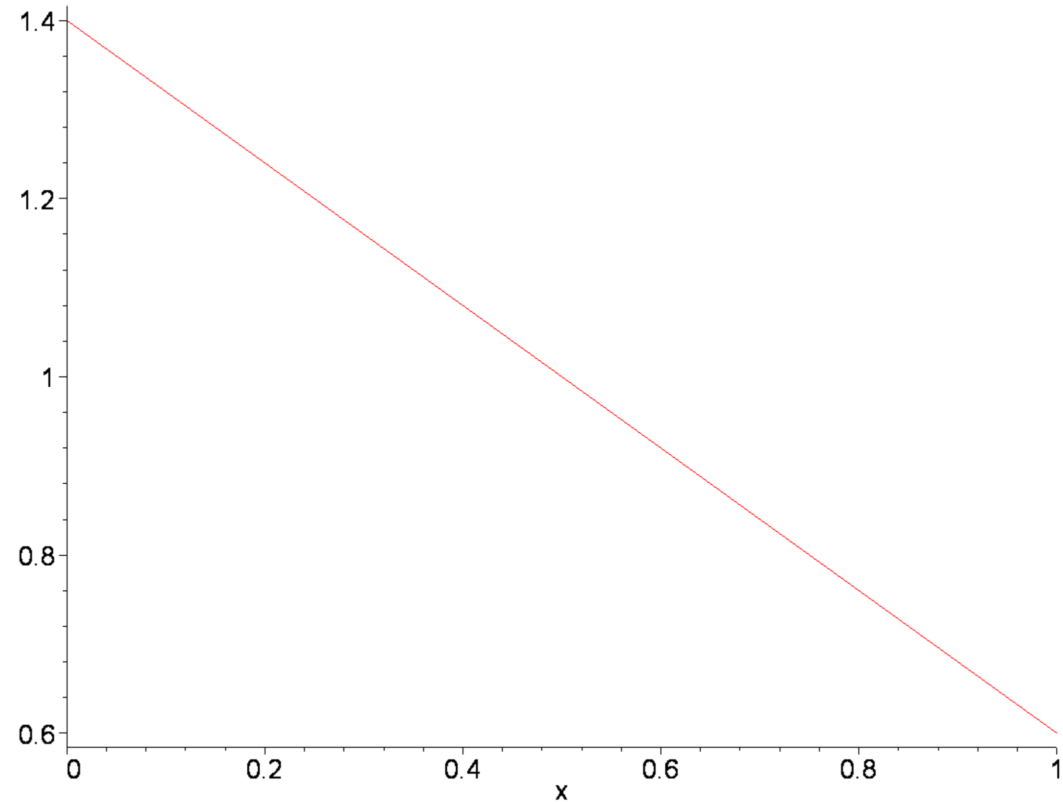
$$a\_2 := 0$$

Como o coeficiente a2=0 então o polinômio Q2(x) de fato é um polinômio de grau 1 e igual ao polinômio Q1(x)

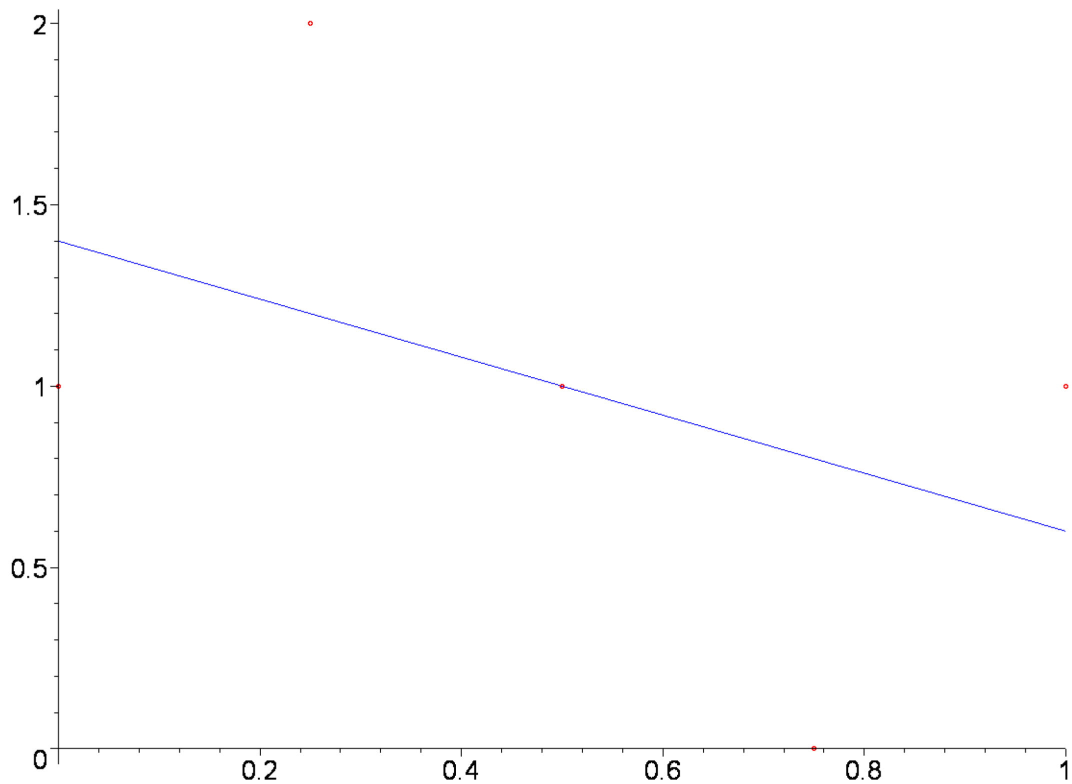
```
> Q_2:=x->a_0+a_1*P_1(x)+a_2*P_2(x);
```

$$Q\_2 := x \rightarrow a\_0 + a\_1 \; \mathrm{P\_1}(x) + a\_2 \; \mathrm{P\_2}(x)$$

```
> zb:=plot([Q_2(x)], x=0...1.0, color=[blue], style=[line]):
> plot([Q_2(x)], x=0...1.0, color=[red], style=[line]);
```




```
> display(plot([ZZ1],style=[point],symbol=circle),z4,zb);
```



POLINÔMIO DE GRAU 3

USANDO POLINÔMIOS ORTOGONAIS

Recorrência: P_{i+1}=(x-alpha{i+1}P_{i}+beta_i P_{i-1}

```
> P_0:= x->1;
```

$$P\_0 := 1$$

```
> P_1:=x-> x-ALPHA1;
```

$$P\_1 := x \to x - ALPHA1$$

OS POLINÔMIOS (P_0, P_1, P_2)SÃO ORTOGONAIS COM RELAÇÃO AOS 5 PONTOS DEFINIDOS

```
> P_2:=x ->(x-ALPHA2)*P_1(x)-BETA1*P_0(x);
```

$$P\_2 := x \to (x - ALPHA2)\, \mathrm{P\_1}(x) - BETA1\, \mathrm{P\_0}(x)$$

```
> P_2(x):=(x-ALPHA2)*P_1(x)-BETA1*P_0(x);
```

$$\mathrm{P\_2}(x) := (x - .5000000000)^2 - .1250000000$$

```
> BETA2:=0:;
> BETA3:=0:;
> ALPHA3:=0:;
> ALPHA4:=0:;
> for i from 1 to N do
> BETA2:=BETA2+(x[i]*(P_2(x[i]))*(P_1(x[i])));
> BETA3:=BETA3+(P_1(x[i]))^2;
> ALPHA3:=ALPHA3+(x[i]*(P_2(x[i]))^2);
> ALPHA4:=ALPHA4+ (P_2(x[i])^2);
> od:;
> BETA4:=BETA2/BETA3;
```

$$BETA4 := .08750000000$$

```
> ALPHA5:=ALPHA3/ALPHA4;
```

$$ALPHA5 := .5000000000$$

```
>
> P_3:=x-> (x-ALPHA5)*P_2(x)-BETA4*P_1(x);
```

$$P\_3 := x \to (x - ALPHA5)\, \mathrm{P\_2}(x) - BETA4\, \mathrm{P\_1}(x)$$

```
> P_3(x):=(x-ALPHA5)*P_2(x)-BETA4*P_1(x);
```

$$\mathrm{P\_3}(x) := (x - .5000000000)\,((x - .5000000000)^2 - .1250000000) - .08750000000\, x + .04375000000$$

```
> a44:=0:
> b4:=0:
> for i from 1 to N do
> a44:=a44+(P_3(x[i])^2):
> b4:=b4+f[i]*(P_3(x[i]));
> od:;
> a3:=b4/a44;
```

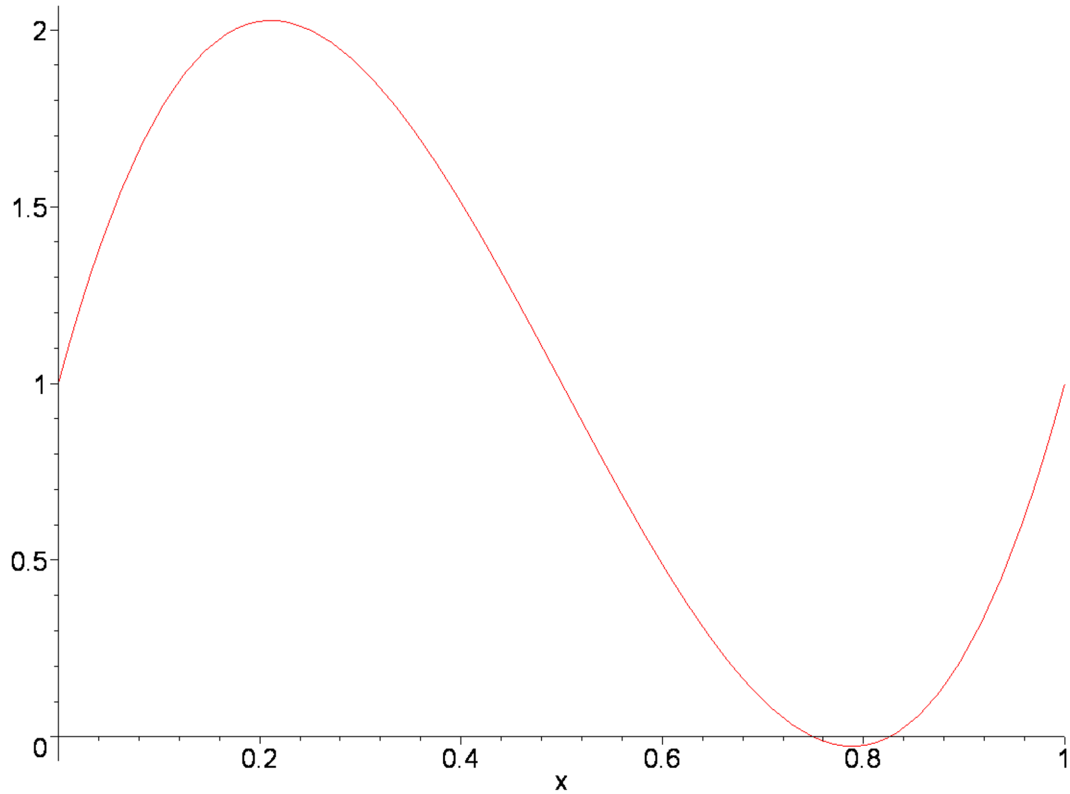
$$a3 := 21.33333333$$

```
> Q3:=x-> a_0+a_1*P_1(x)+a_2*P_2(x)+a3*P_3(x):;
>
> Q_3(x):=a_0+a_1*P_1(x)+a_2*P_2(x)+a3*P_3(x);
```

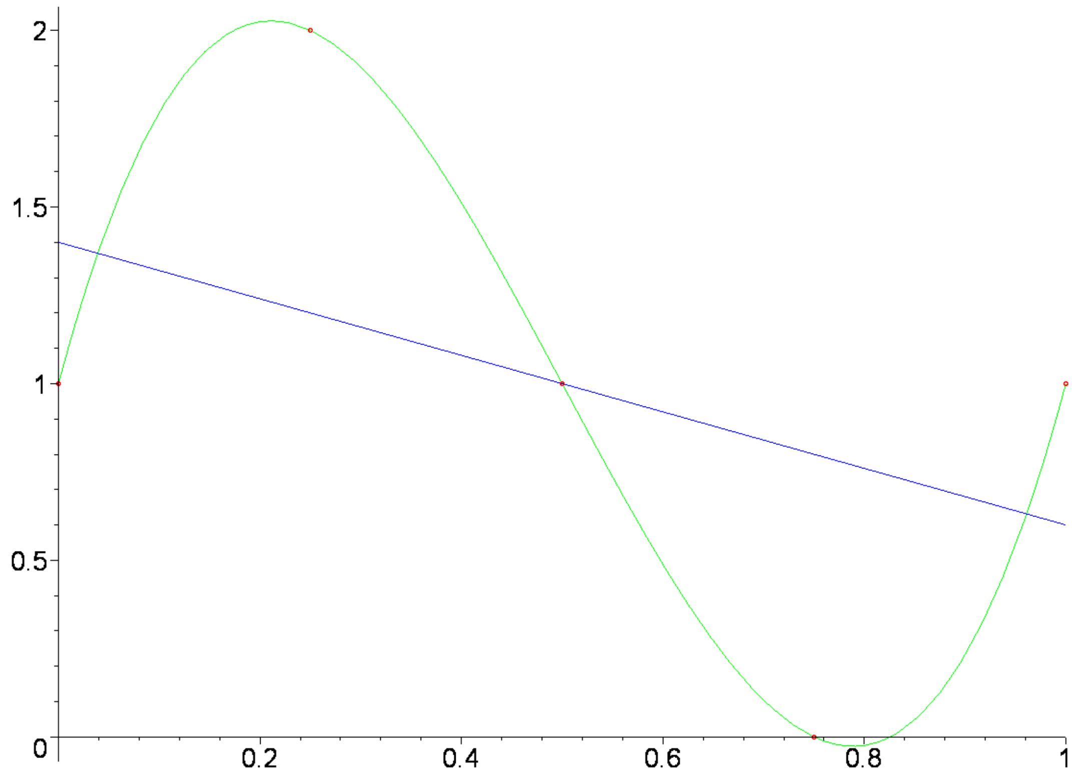
$$Q\_3(x) := 2.333333333 - 2.666666666\, x + 21.33333333\,(x - .5000000000)\,((x - .5000000000)^2 - .1250000000)$$

Esse é o polinômio de grau 3, obtido pela base de polinômios Ortogonais, obtido pelo Método de Mínimos Quadrados.

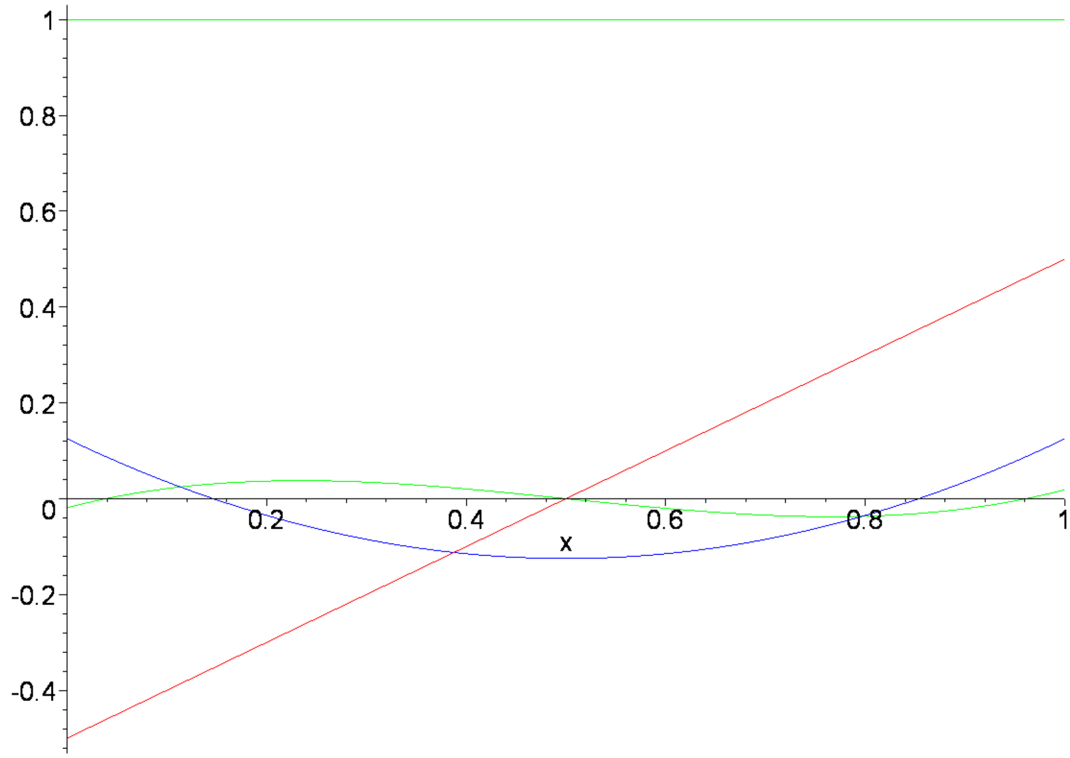
```
> zb3:=plot([Q3(x)], x=0.0..1.0, color=[green], style=[line]):
> plot([Q3(x)], x=0.0..1.0, color=[red], style=[line]);
```




```
> display(plot([ZZ1],style=[point],symbol=circle),z4,zb,zb3);
```




```
> p11:=plot([P_0(x)], x=0.0..1.0, color=[green], style=[line]):
> p12:=plot([P_1(x)], x=0.0..1.0, color=[red], style=[line]):
> p13:=plot([P_2(x)], x=0.0..1.0, color=[blue], style=[line]):
> p14:=plot([P_3(x)], x=0.0..1.0, color=[green], style=[line]):
> display(p11,p12,p13,p14);
```



Os graficos mostrar a ortogonalidade dos polinomios P_0, P_1, P_2, P_3

EXEMPLO 2: POLINÔMIO ORTOGONAIS

POLINÔMIOS ORTOGONAIS

Ajuste de curvas por polinomios de grau 1 e 2.

Recorrência: P_{i+1}=(x-alpha{i+1}P_{i}+beta_i P_{i-1}

POLINÔMIOS DE GRAU 1

```
> x[1]:=0.5:;
> x[2]:=0.7:
> x[3]:=0.9:
> x[4]:=1.1:
> x[5]:=1.5:
> x[6]:=1.7:
> x[7]:=2.1:
> x[8]:=2.9:
> f[1]:=0.24:;
> f[2]:=0.75:
> f[3]:=1.25:
> f[4]:=1.5:
> f[5]:=2.9:
```

```
> f[6]:=3.20:
> f[7]:=4.15:
> f[8]:=5.4:
> N:=8:
> P_0:= x->1;
```

$$P\_0 := 1$$

```
> ALPHA1:=0;
```

$$ALPHA1 := 0$$

```
> ALPHA1:=ALPHA1+(1/N)*(x[1]+x[2]+x[3]+x[4]+x[5]+x[6]+x[7]+x[8]);
```

$$ALPHA1 := 1.425000000$$

```
> P_1:=x->x-ALPHA1;
```

$$P\_1 := x \rightarrow x - ALPHA1$$

```
> a11:=0:
> a22:=0:
> b1:=0:
> b2:=0:
> for i from 1 to N do
> a11:=a11 +1;
> a22:=a22+(P_1(x[i]))^2;
> b1:=b1+f[i]:
> b2:=b2+f[i]*P_1(x[i]):
> od:;
> a_0:=b1/a11;
```

$$a\_0 := 2.423750000$$

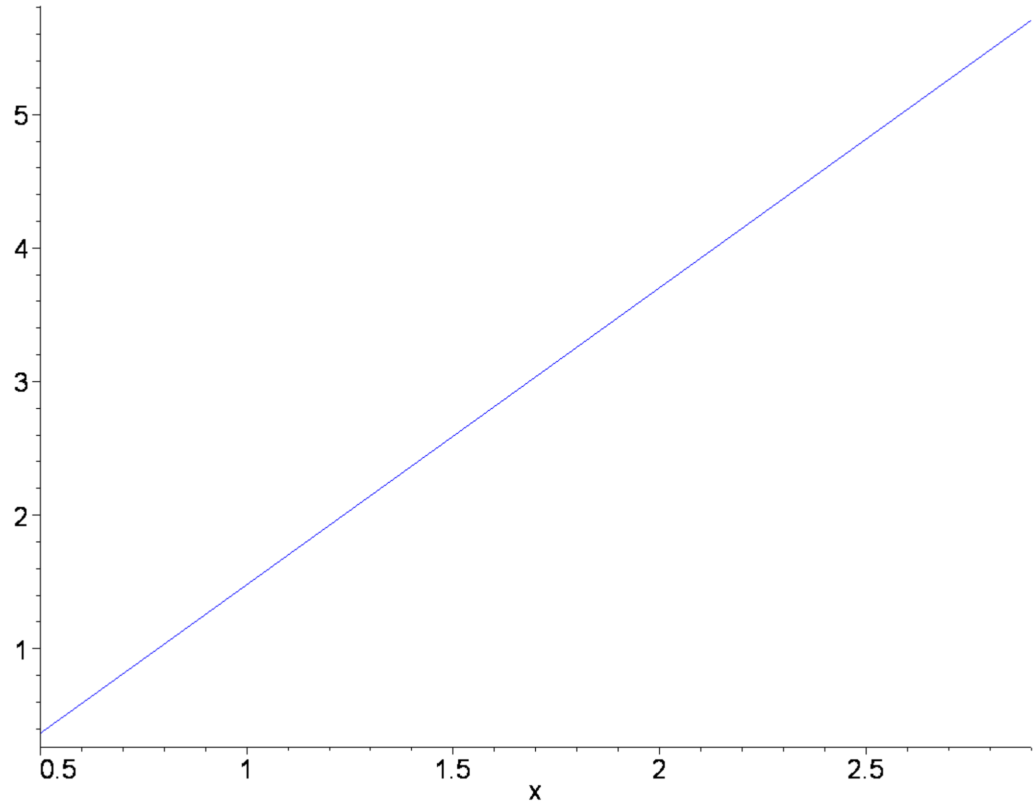
```
> a_1:=b2/a22;
```

$$a\_1 := 2.224413408$$

```
> Q1(x):=a_0+a_1*P_1(x);
```

$$\mathrm{Q1}(x) := -.746039106 + 2.224413408\, x$$

```
> z4:=plot([Q1(x)], x=0.5..2.9, color=[blue], style=[line]):
> plot([Q1(x)], x=0.5..2.9, color=[blue], style=[line]);
```




```
> L:= vector([f[1],f[2], f[3], f[4],f[5],f[6],f[7],f[8]]);
```

$L := [.24, .75, 1.25, 1.5, 2.9, 3.20, 4.15, 5.4]$

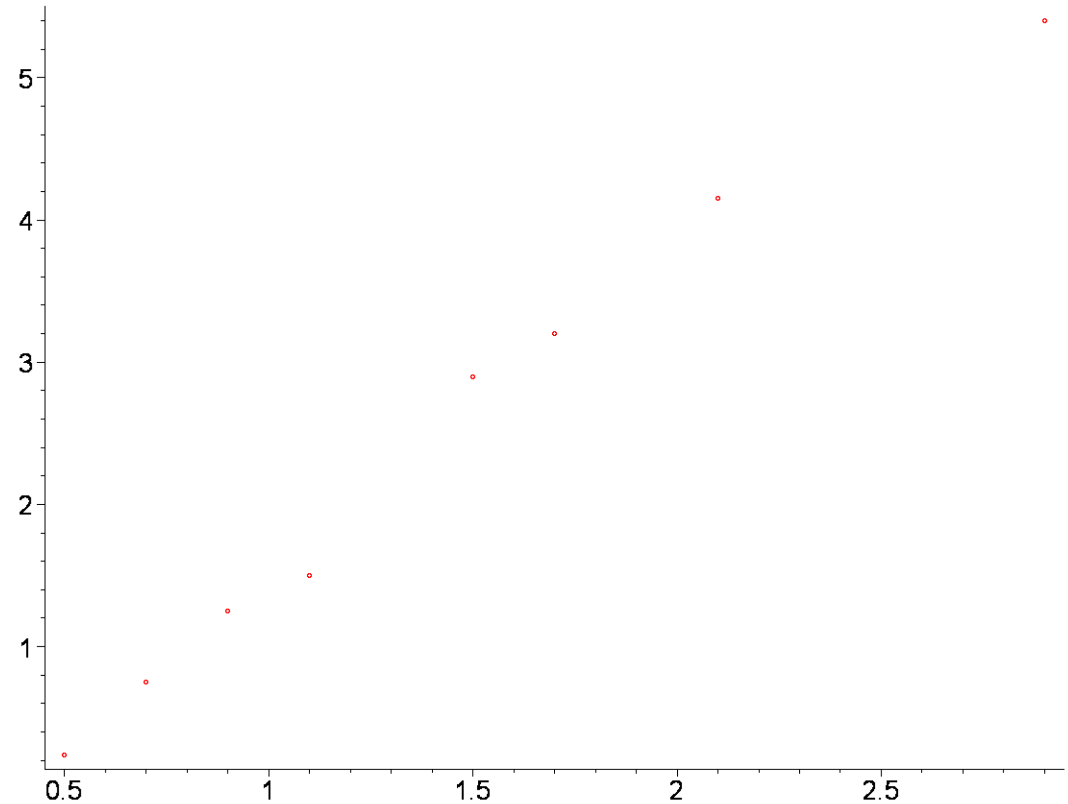
```
> X:=vector([x[1],x[2],x[3],x[4],x[5],x[6],x[7],x[8]]);
```

$X := [.5, .7, .9, 1.1, 1.5, 1.7, 2.1, 2.9]$

```
> ZZ:= [[ X[j], L[j]] $j=1..8];
```

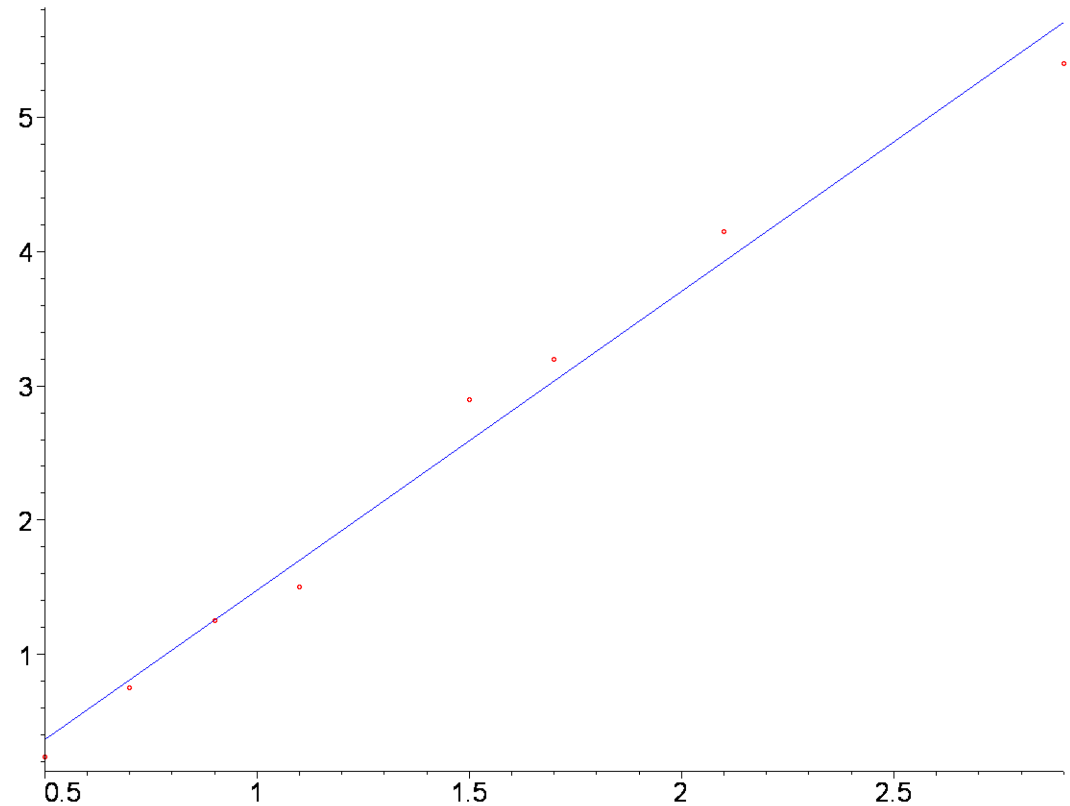
$ZZ := [[.5, .24], [.7, .75], [.9, 1.25], [1.1, 1.5], [1.5, 2.9], [1.7, 3.20], [2.1, 4.15], [2.9, 5.4]]$

```
> plot(ZZ,style=point,symbol=circle);
```



$ZZ1 := [[0, 1.0], [.25, 2.0], [.5, 1.0], [.75, 0], [1.0, 1.0]]$

```
> display(plot([ZZ],style=[point],symbol=circle),z4);
```



POLINÔMIO DE GRAU 2

USANDO POLINÔMIOS ORTOGONAIS

Recorrência: P_{i+1}=(x-alpha{i+1}P_{i}+beta_i P_{i-1}

```
> BETA0:=0:;
> gama0:=0:;
> gama1:=0:;
> Q1 := x ->a_0+a_1*P_1(x);
```

$$Q1 := x \rightarrow a\_0 + a\_1 \, \mathrm{P\_1}(x)$$

```
> for i from 1 to N do
> BETA0:=BETA0+x[i]*(P_1(x[i]));
> gama0:=gama0+(x[i]*(P_1(x[i])^2));
> gama1:=gama1+(P_1(x[i])^2):
> od:;
> BETA1:=(1/N)*BETA0;
```

$$BETA1 := .5593750000$$

```
> ALPHA2:=gama0/gama1;
```

$$ALPHA2 := 1.913547486$$

```
> P_2:=x ->(x-ALPHA2)*P_1(x)-BETA1*P_0(x);
```

$$P\_2 := x \rightarrow (x - ALPHA2) \, \mathrm{P\_1}(x) - BETA1 \, \mathrm{P\_0}(x)$$

```
> a33:=0:
> b3:=0:
> for i from 1 to N do
> a33:=a33+(P_2(x[i]))^2:
> b3:=b3+f[i]*P_2(x[i]):
> od:;
>
> a_2:=b3/a33;
```

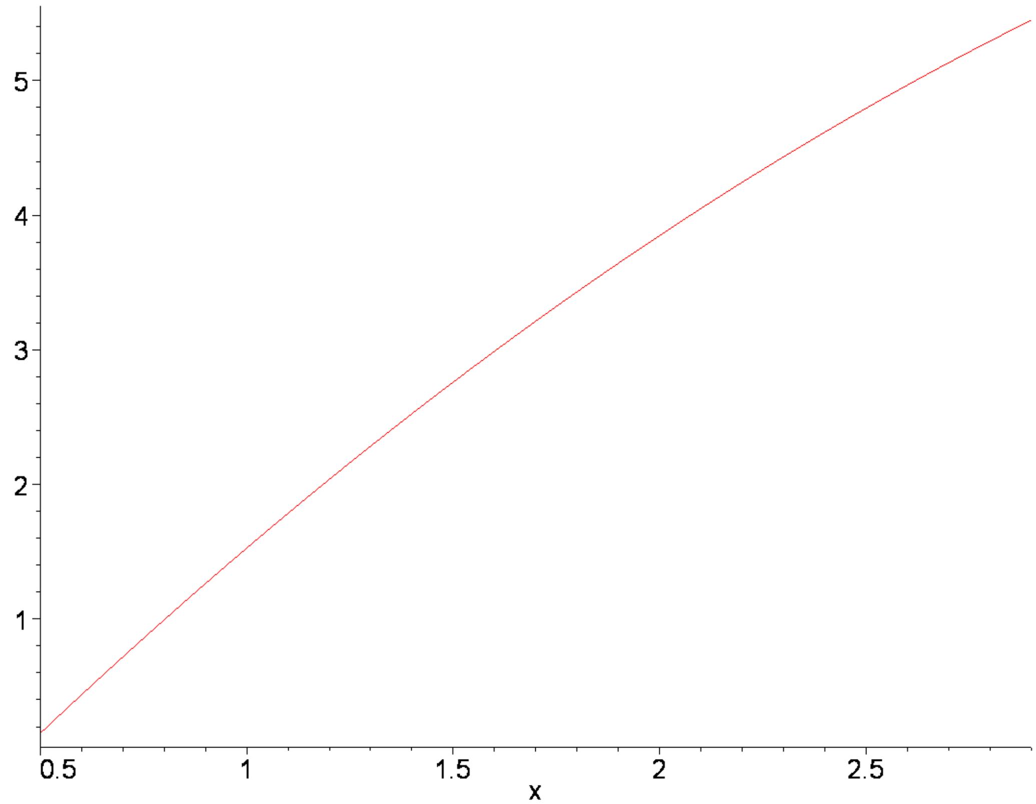
$$a\_2 := -.2870760258$$

Como o coeficiente a2=0 então o polinômio Q2(x) de fato é um polinômio de grau 1 e igual ao polinômio Q1(x)

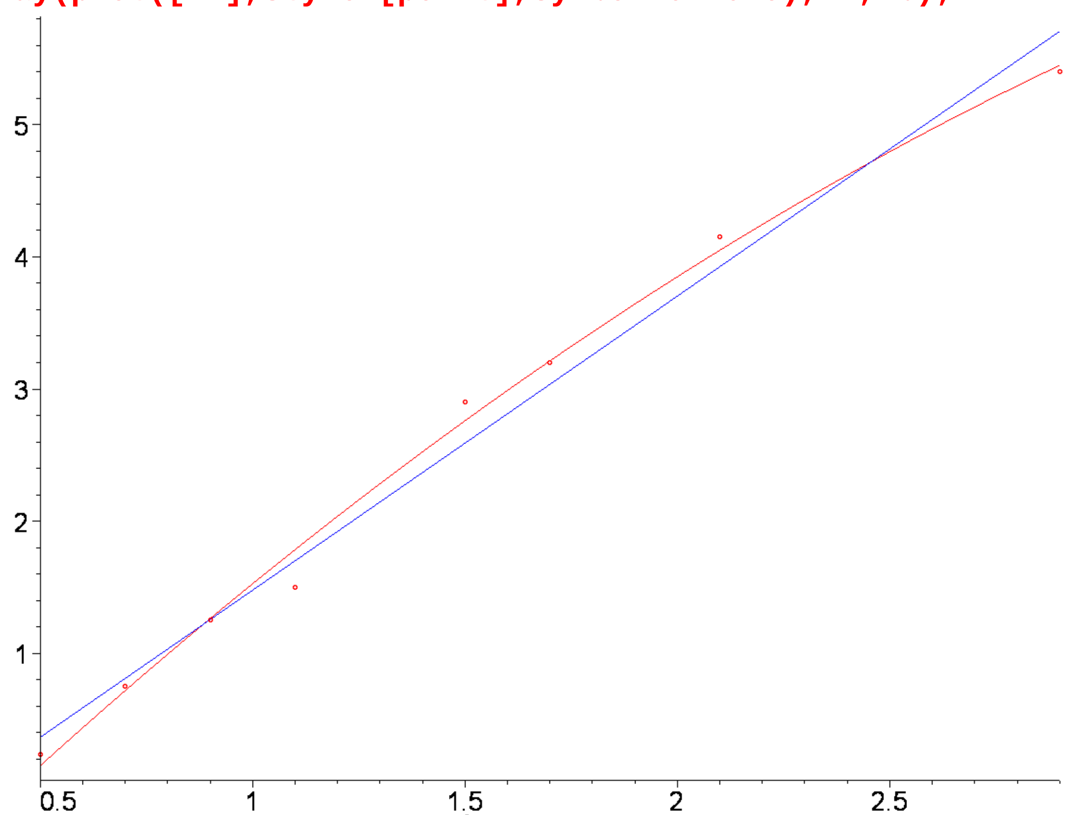
```
> Q_2:=x->a_0+a_1*P_1(x)+a_2*P_2(x);
```

$$Q\_2 := x \rightarrow a\_0 + a\_1 \, \mathrm{P\_1}(x) + a\_2 \, \mathrm{P\_2}(x)$$

```
> zb:=plot([Q_2(x)], x=0.5...2.9, color=[red], style=[line]):
> plot([Q_2(x)], x=0.5...2.9, color=[red], style=[line]);
```




```
> display(plot([ZZ],style=[point],symbol=circle),z4,zb);
```



POLINÔMIO DE GRAU 3

USANDO POLINÔMIOS ORTOGONAIS

Recorrência: P_{i+1}=(x-alpha{i+1}P_{i}+beta_i P_{i-1}

```
> P_0:= x->1:;
> P_1:=x-> x-ALPHA1:;
```

OS POLINÔMIOS (P_0, P_1, P_2)SÃO ORTOGONAIS COM RELAÇÃO AOS 5 PONTOS DEFINIDOS

```
> P_2:=x ->(x-ALPHA2)*P_1(x)-BETA1*P_0(x):;
> P_2(x):=(x-ALPHA2)*P_1(x)-BETA1*P_0(x);
```

$$P\_2(x) := (x - 1.913547486)(x - 1.425000000) - .5593750000$$

```
> BETA2:=0:;
> BETA3:=0:;
> ALPHA3:=0:;
> ALPHA4:=0:;
> for i from 1 to N do
> BETA2:=BETA2+(x[i]*(P_2(x[i]))*(P_1(x[i])));
> BETA3:=BETA3+(P_1(x[i]))^2;
> ALPHA3:=ALPHA3+(x[i]*(P_2(x[i]))^2);
> ALPHA4:=ALPHA4+ (P_2(x[i])^2);
> od:;
> BETA4:=BETA2/BETA3;
```

$$BETA4 := .5521565493$$

```
> ALPHA5:=ALPHA3/ALPHA4;
```

$$ALPHA5 := 1.757024661$$

```
> P_3:=x-> (x-ALPHA5)*P_2(x)-BETA4*P_1(x);
```

$$P\_3 := x \to (x - ALPHA5)\,P\_2(x) - BETA4\,P\_1(x)$$

```
> P_3(x):=(x-ALPHA5)*P_2(x)-BETA4*P_1(x);
```

$$P\_3(x) := (x - 1.757024661)((x - 1.913547486)(x - 1.425000000) - .5593750000) - .5521565493\,x + .7868230828$$

```
> a44:=0:
> b4:=0:
> for i from 1 to N do
> a44:=a44+(P_3(x[i])^2):
> b4:=b4+f[i]*(P_3(x[i]));
> od:;
> a3:=b4/a44;
```

$$a3 := -.2507173731$$

```
> Q3:=x-> a_0+a_1*P_1(x)+a_2*P_2(x)+a3*P_3(x);
```

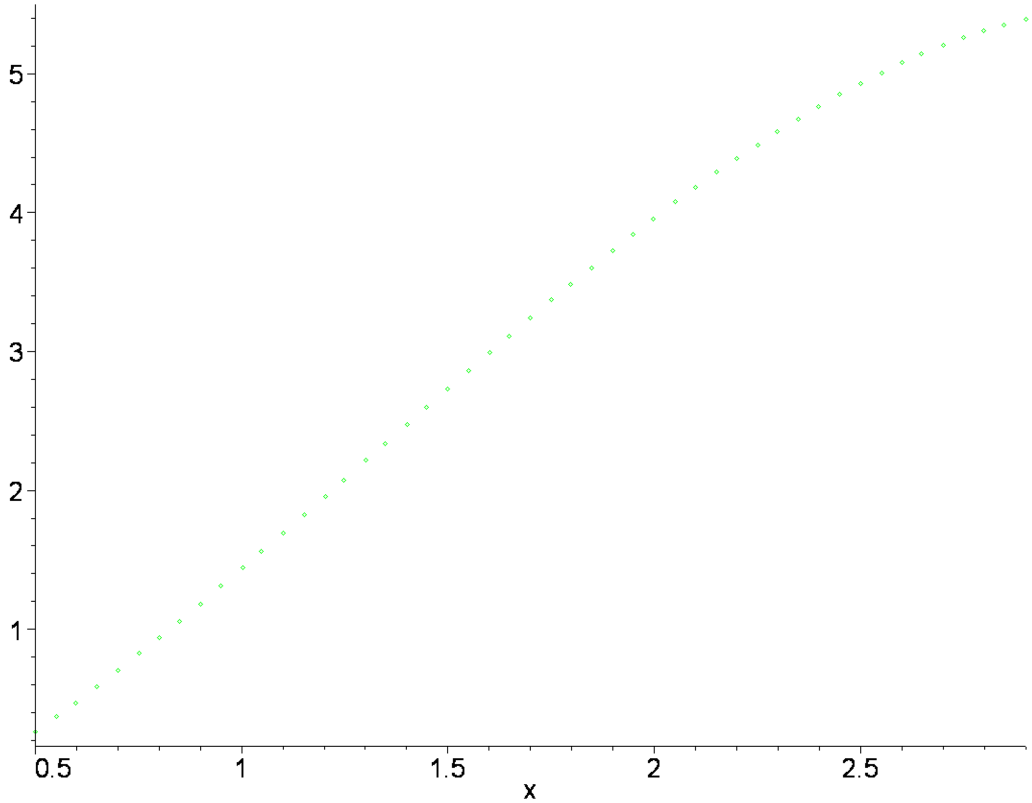
$$Q3 := x \to a\_0 + a\_1\,P\_1(x) + a\_2\,P\_2(x) + a3\,P\_3(x)$$

```
> Q_3(x):=a_0+a_1*P_1(x)+a_2*P_2(x)+a3*P_3(x);
```

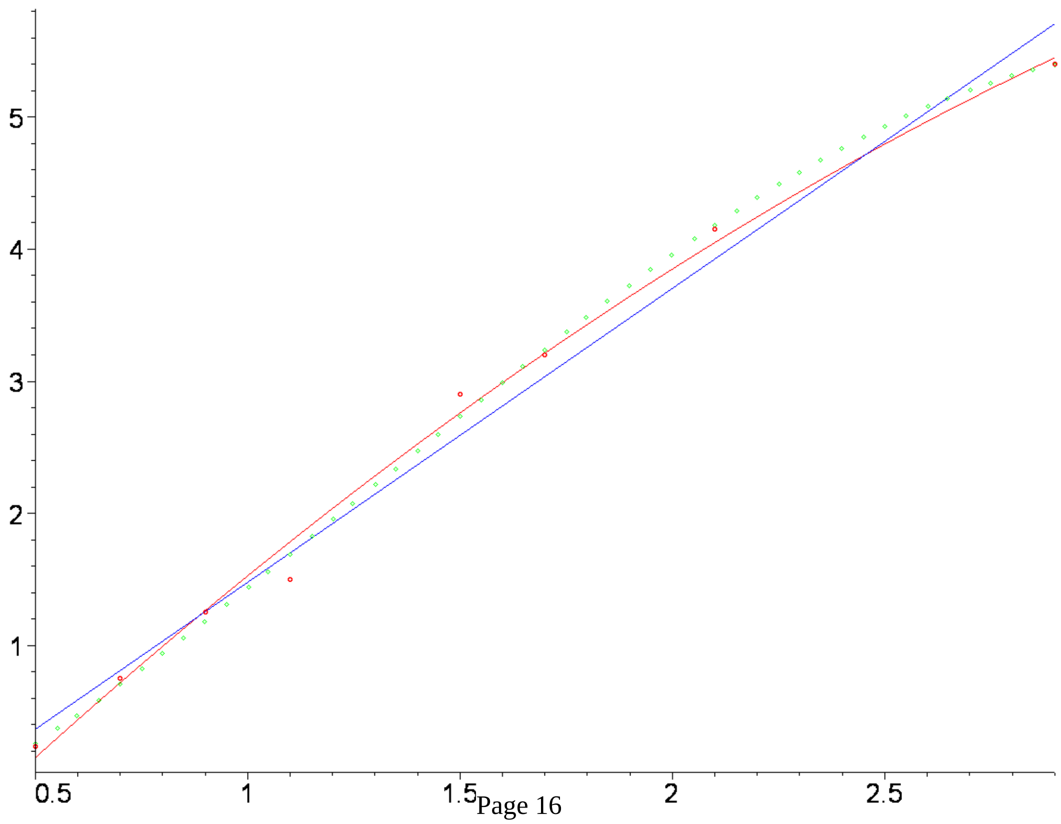
$$Q\_3(x) := -.7827261705 + 2.362848648\,x - .2870760258\,(x - 1.913547486)(x - 1.425000000) - .2507173731\,(x - 1.757024661)((x - 1.913547486)(x - 1.425000000) - .5593750000)$$

Esse é o polinômio de grau 3, obtido pela base de polinômios Ortogonais, obtido pelo Método de Mínimos Quadrados.

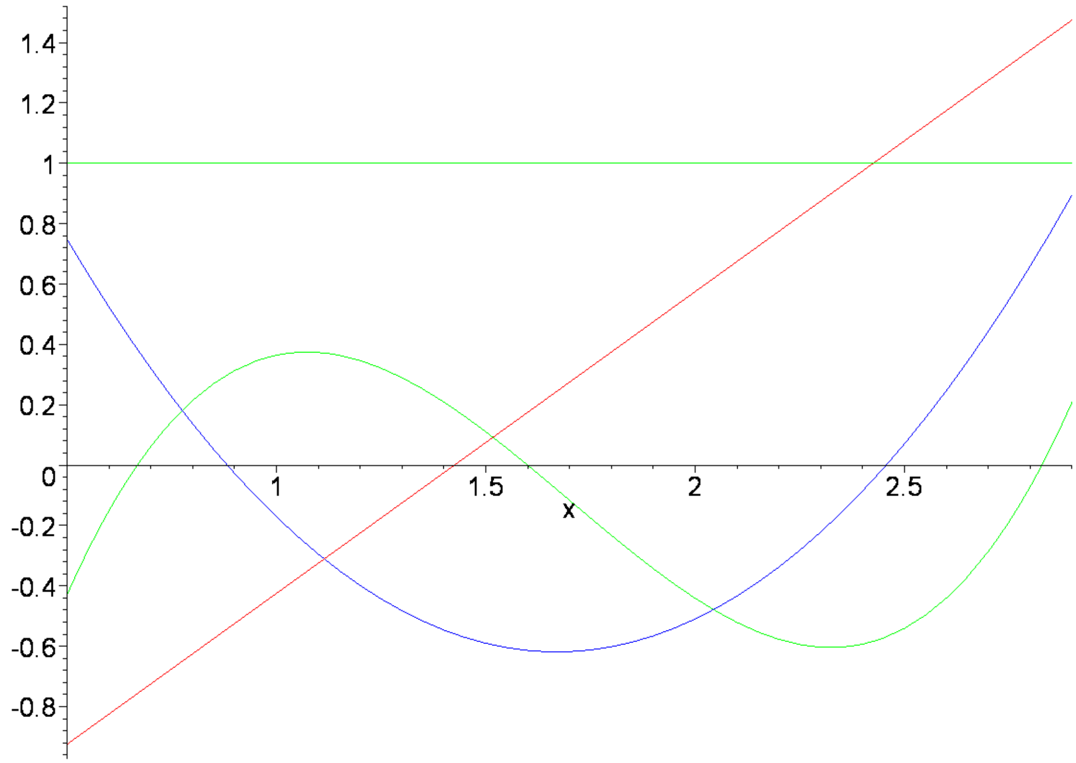
```
> zb3:=plot([Q3(x)], x=0.5..2.9, color=[green], style=[point]):
> plot([Q3(x)], x=0.5..2.9, color=[green], style=[point]);
```




```
> display(plot([ZZ],style=[point],symbol=circle),z4,zb,zb3);
```



```
> q11:=plot([P_0(x)], x=0.5..2.9, color=[green], style=[line]):
> q12:=plot([P_1(x)], x=0.5..2.9, color=[red], style=[line]):
> q13:=plot([P_2(x)], x=0.5..2.9, color=[blue], style=[line]):
> q14:=plot([P_3(x)], x=0.5..2.9, color=[green], style=[line]):
> display(q11,q12,q13,q14);
```




```
>
>
>
>
>
>
>
```

Os graficos mostrar a ortogonalidade dos polinomios P_0, P_1, P_2, P_3