```
> restart:
> with(plots):
> with(linalg):
Warning, new definition for norm
Warning, new definition for trace
```

POLINÔMIOS POR PARTES

PONTOS CONHECIDOS

```
> x0:=-1:
> x1:=-0.5:
> x2:=0.0:
> x3:=0.5:
> x4:=1.0:
```

VALOR DA FUNÇÃO

```
> y0:=0:
> y1:=3:
> y2:=-1:
> y3:=1:
> y4:=0:
> h:=0.5:
> X[0]:= array(1..5,[x0,x1,x2,x3,x4]):;
> f[x0,x1]:=(y1-y0)/h;
```

$$f_{-1,\,-.5} := 6.000000000$$

```
> f[x1,x2]:=(y2-y1)/h;
```

$$f_{-.5,\,0} := -8.000000000$$

```
> f[x2,x3]:=(y3-y2)/h;
```

$$f_{0,\,.5} := 4.000000000$$

```
> f[x3,x4]:=(y4-y3)/h;
```

$$f_{.5,\,1.0} := -2.000000000$$

```
> f[x0,x1,x2]:=(f[x1,x2]-f[x0,x1])/(2*h);
```

$$f_{-1,\,-.5,\,0} := -14.00000000$$

```
> f[x1,x2,x3]:=(f[x2,x3]-f[x1,x2])/(2*h);
```

$$f_{-.5,\,0,\,.5} := 12.00000000$$

```
> f[x2,x3,x4]:=(f[x3,x4]-f[x2,x3])/(2*h);
```

$$f_{0,\,.5,\,1.0} := -6.000000000$$

```
> f[x0,x1,x2,x3]:=(f[x1,x2,x3]-f[x0,x1,x2])/(3*h);
```

$$f_{-1,\,-.5,\,0,\,.5} := 17.33333333$$

```
> f[x1,x2,x3,x4]:=(f[x2,x3,x4]-f[x1,x2,x3])/(3*h);
```

$$f_{-.5,\,0,\,.5,\,1.0} := -12.00000000$$

```
> f[x0,x1,x2,x3,x4]:=(f[x1,x2,x3,x4]-f[x0,x1,x2,x3])/(4*h);
```

$$f_{-1,\,-.5,\,0,\,.5,\,1.0} := -14.66666667$$

POLINÔMIOS DE GRAU 1

```
> P_11:=y0+f[x0,x1]*(x-x0);
```

$$P\_11 := 6.000000000\,x + 6.000000000$$

```
> P_12:=y1+f[x1,x2]*(x-x1);
```

$$P\_12 := -1.000000000 - 8.000000000\,x$$

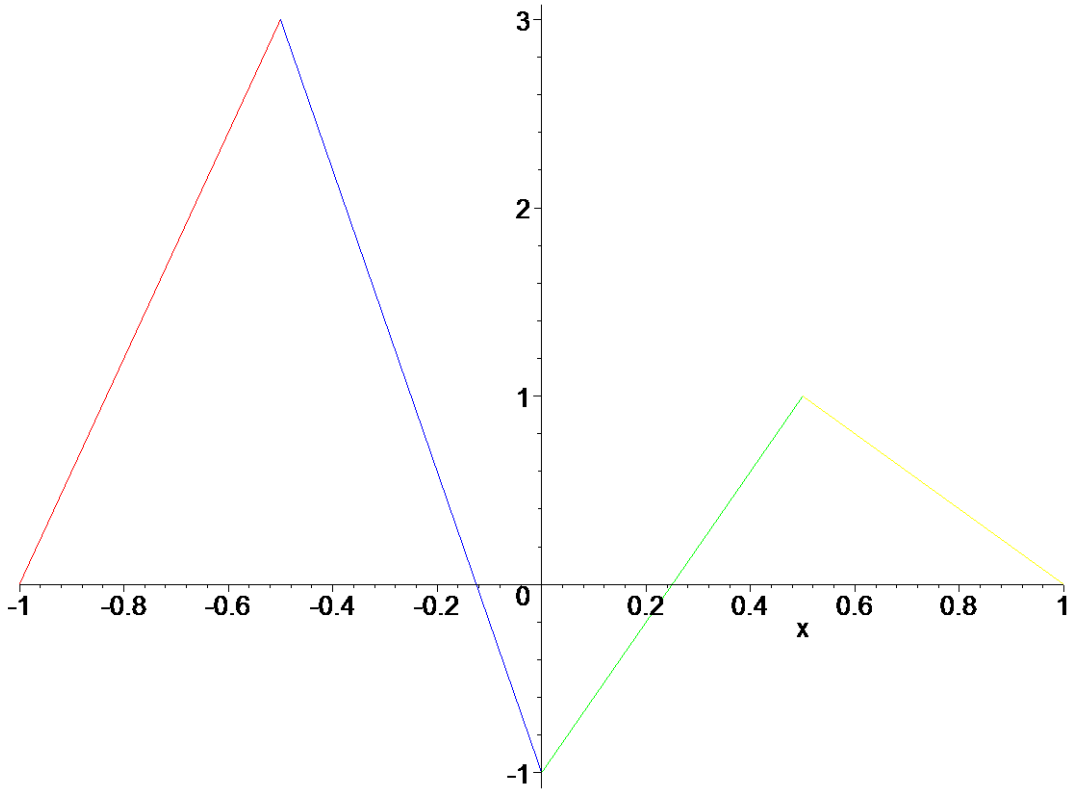
```
> P_13:=y2+f[x2,x3]*(x-x2);
```

$$P\_13 := -1 + 4.000000000\,x$$

```
> P_14:=y3+f[x3,x4]*(x-x3);
```

$$P\_14 := 2.000000000 - 2.000000000\,x$$

```
> gr1:=plot(P_11, x=-1..-0.5, color=[red]):
> gr2:=plot(P_12, x=-0.5..0.0, color=[blue]):
> gr3:=plot(P_13, x=0.0..0.5, color=[green]):
> gr4:=plot(P_14, x=0.5..1.0 , color=[yellow]):
> display(gr1,gr2,gr3,gr4);
```



POLINÔMIOS DE GRAU 2(número para de subintervalos)

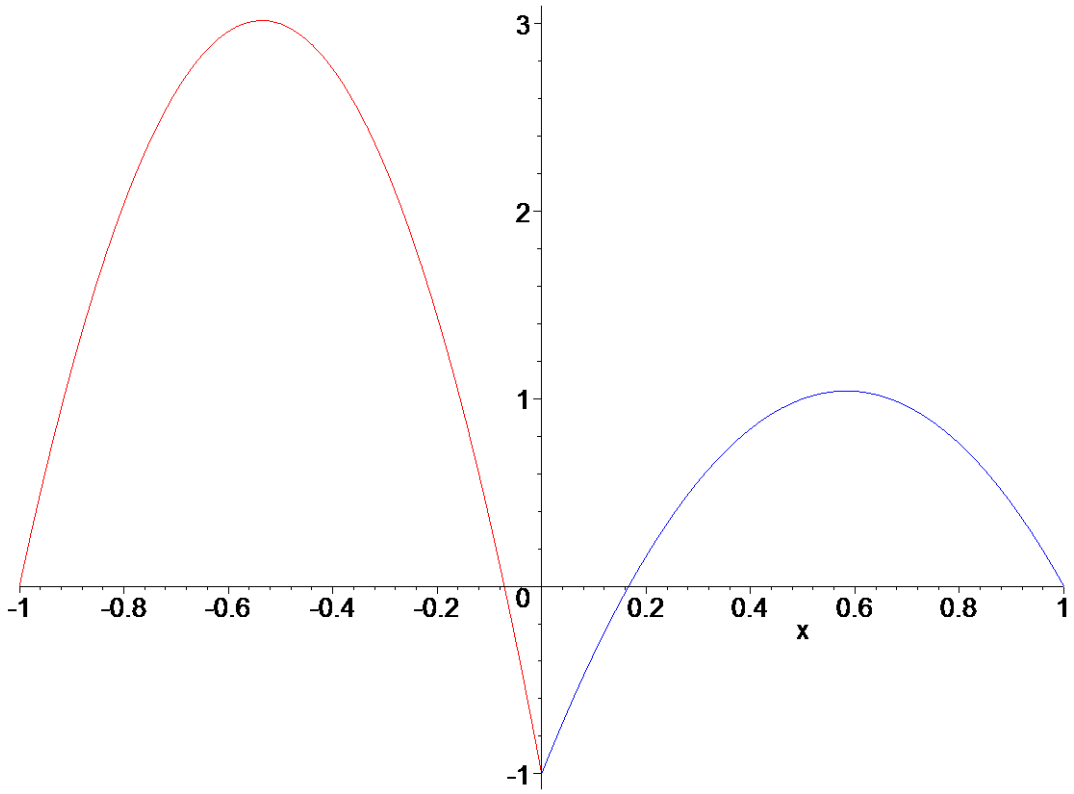
```
> P_21:=y0+f[x0,x1]*(x-x0)+f[x0,x1,x2]*(x-x0)*(x-x1);
```

$$P\_21 := 6.000000000 + 6.000000000\,x - 14.00000000\,(x+1)\,(x+.5)$$

```
> P_22:=y2+f[x2,x3]*(x-x2)+f[x2,x3,x4]*(x-x2)*(x-x3);
```

$$P\_22 := -1 + 4.000000000\,x - 6.000000000\,x\,(x-.5)$$

```
> gr5:=plot(P_21, x=-1.0..0.0, color=[red]):
>
> gr6:=plot(P_22, x=0.0..1.0, color=[blue]):
> display(gr5,gr6);
```

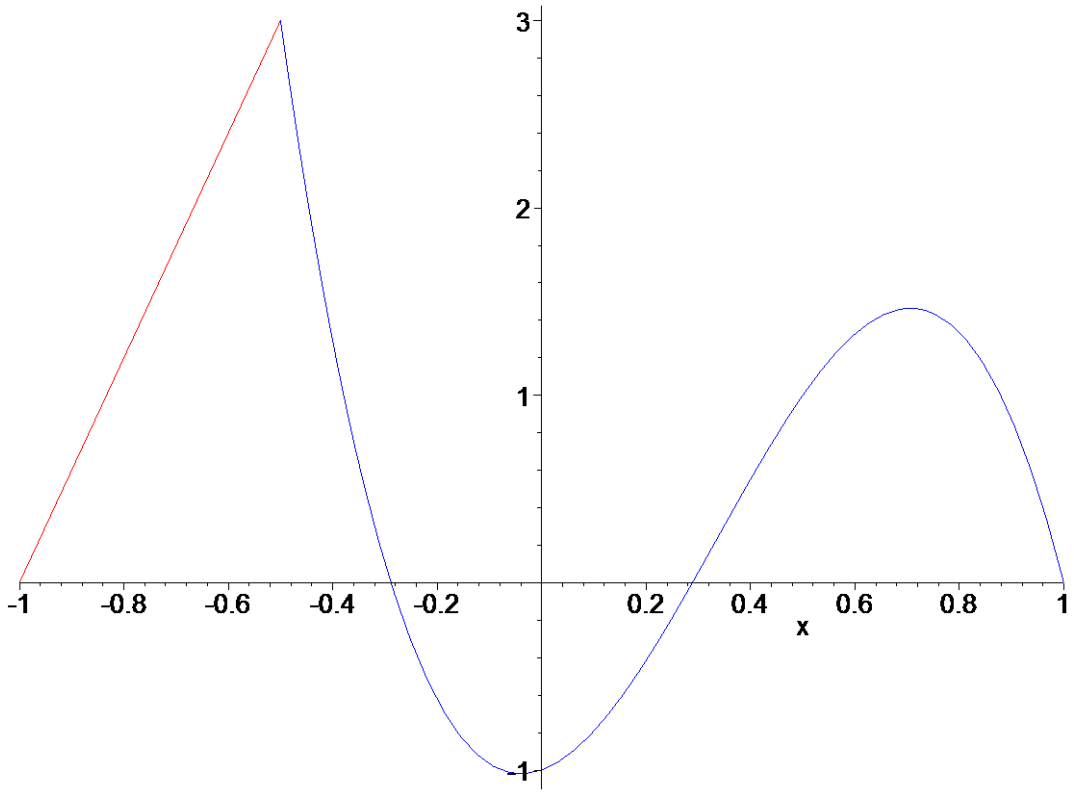


POLINÔMIOS DE GRAU 1 e GRAU 3

```
> P_31:=y1+f[x1,x2]*(x-x1)+f[x1,x2,x3]*(x-x1)*(x-x2)+f[x1,x2,x3,x4
  ]*(x-x1)*(x-x2)*(x-x3);
```

$$P\_31 := -1.000000000 - 8.000000000\,x + 12.00000000\,(x + .5)\,x - 12.00000000\,(x + .5)\,x\,(x - .5)$$

```
> gr7:=plot(P_31, x=-0.5..1.0, color=[blue]):
> display(gr1,gr7);
```

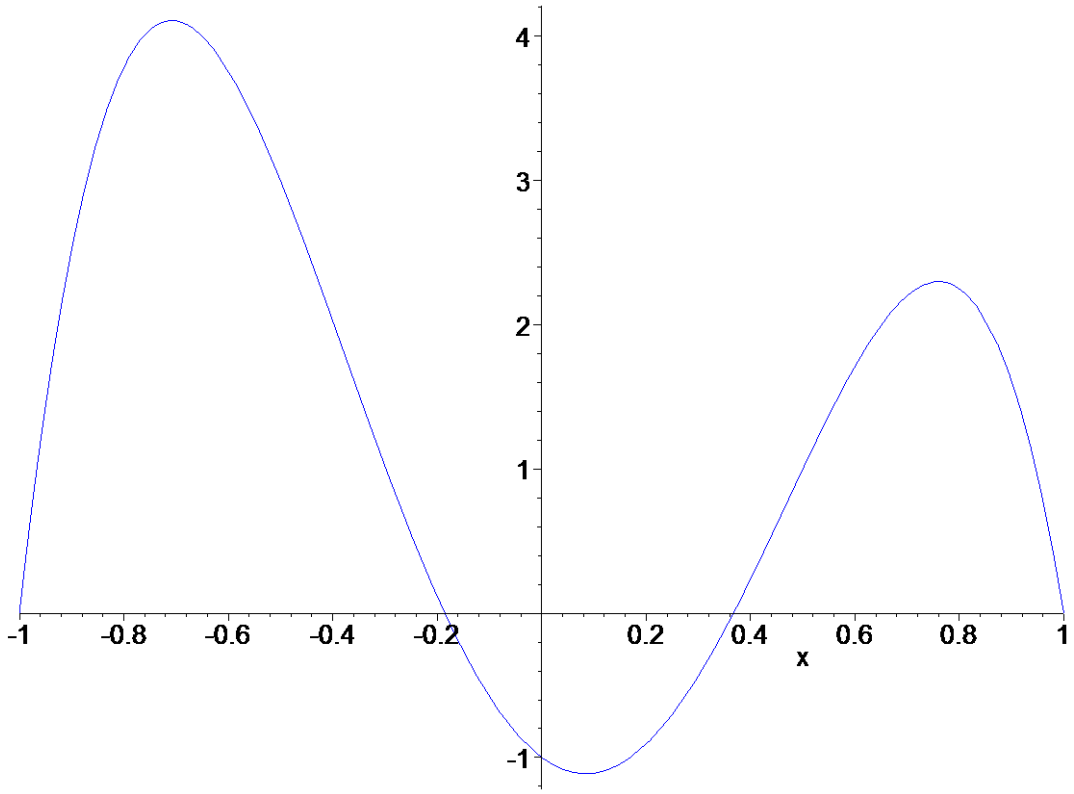


POLINÔMIOS DE GRAU 4

```
> P_41:=y0+f[x0,x1]*(x-x0)+f[x0,x1,x2]*(x-x0)*(x-x1)+f[x0,x1,x2,x3
  ]*(x-x0)*(x-x1)*(x-x2)+f[x0,x1,x2,x3,x4]*(x-x0)*(x-x1)*(x-x2)*(x
  -x3);
```

$$P\_41 := 6.000000000 + 6.000000000\,x - 14.00000000\,(x+1)\,(x+.5) + 17.33333333\,(x+1)\,(x+.5)\,x - 14.66666667\,(x+1)\,(x+.5)\,x\,(x-.5)$$

```
> gr8:=plot(P_41, x=-1.0..1.0, color=[blue]):
> display(gr8);
```



SPLINE CÚBICA

EXEMPLO 3: CINCO PONTOS (QUATRO CURVAS)

```
> with(linalg):
>
> h1:=0.5:
> h2:=0.5:
> h3:=0.5:
> h4:=0.5:
> X[0]:= array(1..5,[x0,x1,x2,x3,x4]):;
> b1:=6*((y2-y1)/h2-(y1-y0)/h1);
```

$$b1 := -84.00000000$$

```
> b2:=6*((y3-y2)/h3-(y2-y1)/h2);
```

$$b2 := 72.00000000$$

```
> b3:=6*((y4-y3)/h4-(y3-y2)/h3);
```

$$b3 := -36.00000000$$

```
> a11:=2*(h1+h2);
```

$$a11 := 2.0$$

```
> a12:=(h2);
```

$$a12 := .5$$

```
> a21:=a12;
```

$$a21 := .5$$

```
> a13:=0:
```

```
> a31:=a13:
> a23:=h3:
> a32:=a23:
> a22:=2*(h2+h3):
> a33:=2*(h3+h4):
>
> A:=matrix(3,3,[[a11,a12,a13],[a21,a22,a23],[a31,a32,a33]]);
```

$$A := \begin{bmatrix} 2.0 & .5 & 0 \\ .5 & 2.0 & .5 \\ 0 & .5 & 2.0 \end{bmatrix}$$

```
> b:=vector(3,[b1,b2,b3]);
```

$$b := [\,-84.00000000,\ 72.00000000,\ -36.00000000\,]$$

```
>
> Y2[2]:=linsolve(A,b);
```

$$Y2_2 := [\,-56.57142857,\ 58.28571428,\ -32.57142857\,]$$

```
> Y0[2]:=0:
> Y1[2]:=Y2[2][1];:
```

$$Y1_2 := -56.57142857$$

```
> Y2[2]:=Y2[2][2];:
```

$$Y2_2 := 58.28571428$$

```
> Y3[2]:=48;
```

$$Y3_2 := 48$$

```
> Y4[2]:=0;:
```

$$Y4_2 := 0$$

```
>
> YY2:= vector(5,[Y0[2],Y1[2],Y2[2],Y3[2],Y4[2]]);
```

$$YY2 := [\,0,\ -56.57142857,\ 58.28571428,\ 48,\ 0\,]$$

YY2 é vetor da segunda derivada

```
> Y0[1]:=(y1-y0)/h1-h1/6*(Y1[2]+2*Y0[2]);
```

$$Y0_1 := 10.71428572$$

```
> Y1[1]:=(y2-y1)/h2-h2/6*(Y2[2]+2*Y1[2]);
```

$$Y1_1 := -3.428571432$$

```
> Y2[1]:=(y3-y2)/h3-h3/6*(Y3[2]+2*Y2[2]);
```

$$Y2_1 := -9.714285720$$

```
> Y3[1]:=(y4-y3)/h4-h4/6*(Y4[2]+2*Y3[2]);
```

$$Y3_1 := -10.00000000$$

```
>
> YY1:=vector(4,[Y0[1],Y1[1],Y2[1],Y3[1]]);
```

$$YY1 := [\,10.71428572,\ -3.428571432,\ -9.714285720,\ -10.00000000\,]$$

```
>
```

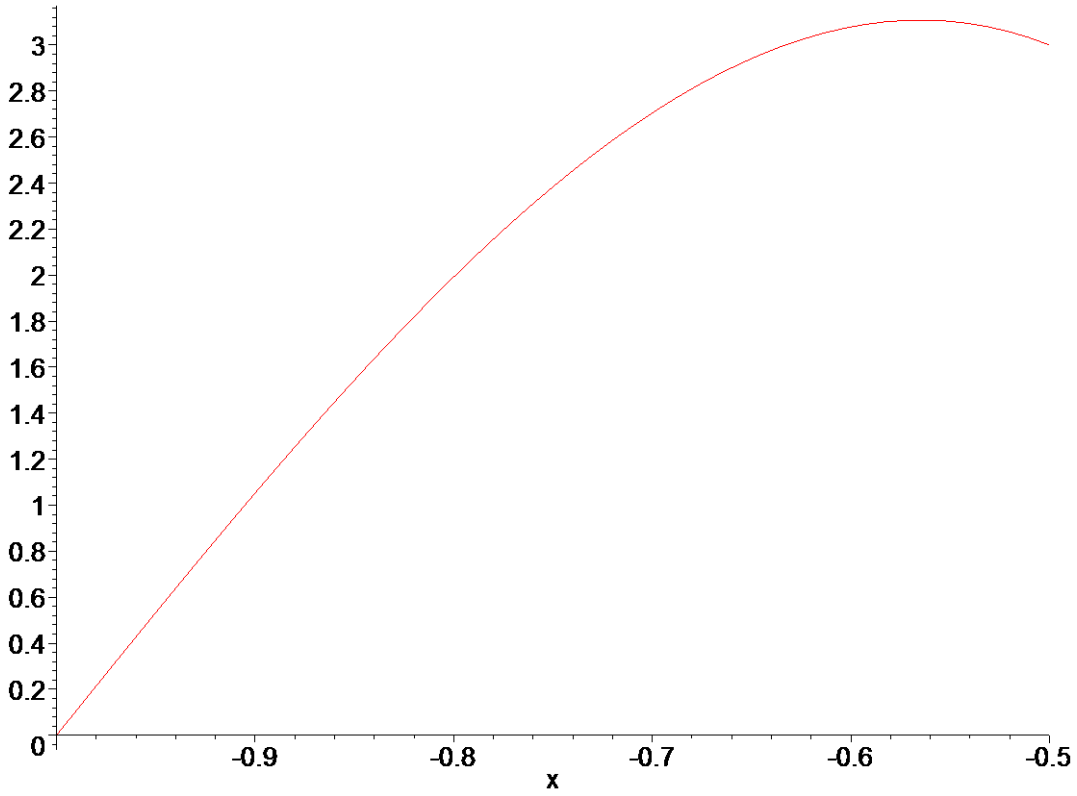
YY1 É A PRIMEIRA DERIVADA

```
> S1:=y0+Y0[1]*(x-x0)+Y0[2]/2*(x-x0)^2+1/(6*h2)*(Y1[2]-Y0[2])*(x-x
```

```
0)^3;
```

$$S1 := 10.71428572 + 10.71428572\,x - 18.85714285\,(x+1)^3$$

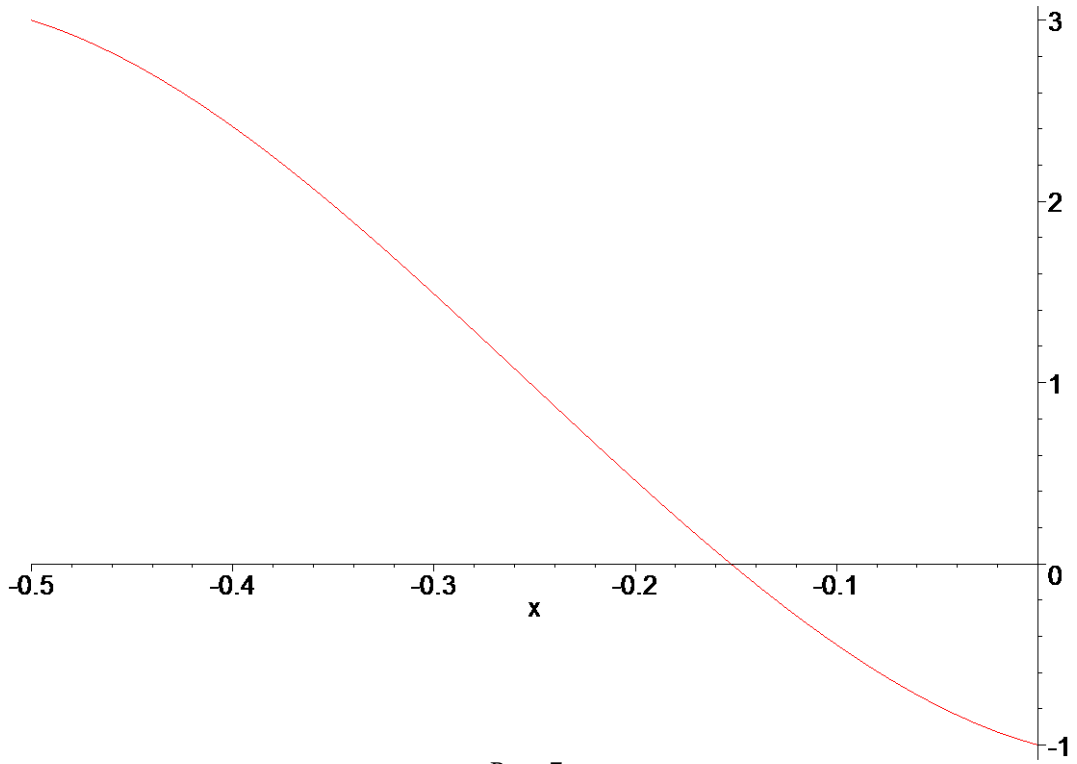
```
> plot([S1(x)], x=-1.0..-0.5, color=[red], style=[line]);
```




```
> S2:=y1+Y1[1]*(x-x1)+Y1[2]/2*(x-x1)^2+1/(6*h2)*(Y2[2]-Y1[2])*(x-x
  1)^3;
```

$$S2 := 1.285714284 - 3.428571432\,x - 28.28571429\,(x+.5)^2 + 38.28571430\,(x+.5)^3$$

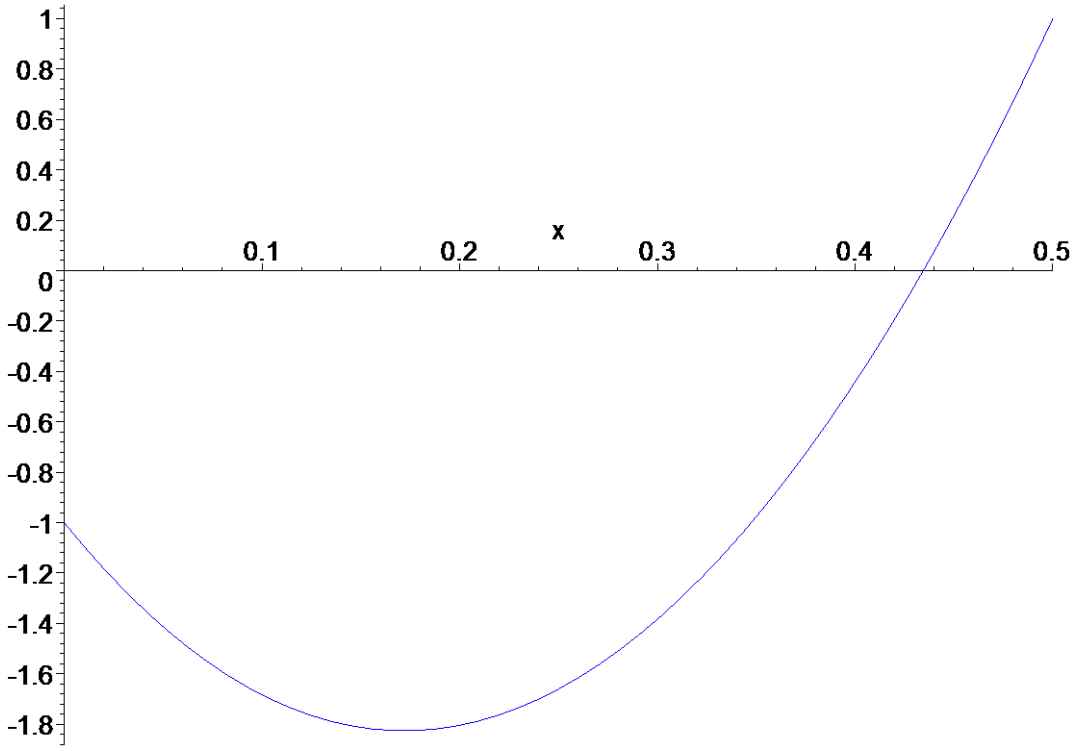
```
> plot([S2(x)], x=-0.5..0.0, color=[red], style=[line]);
```



```
> S3:=y2+Y2[1]*(x-x2)+Y2[2]/2*(x-x2)^2+1/(6*h3)*(Y3[2]-Y2[2])*(x-x
  2)^3;
```

$$S3 := -1 - 9.714285720\,x + 29.14285714\,x^2 - 3.428571427\,x^3$$

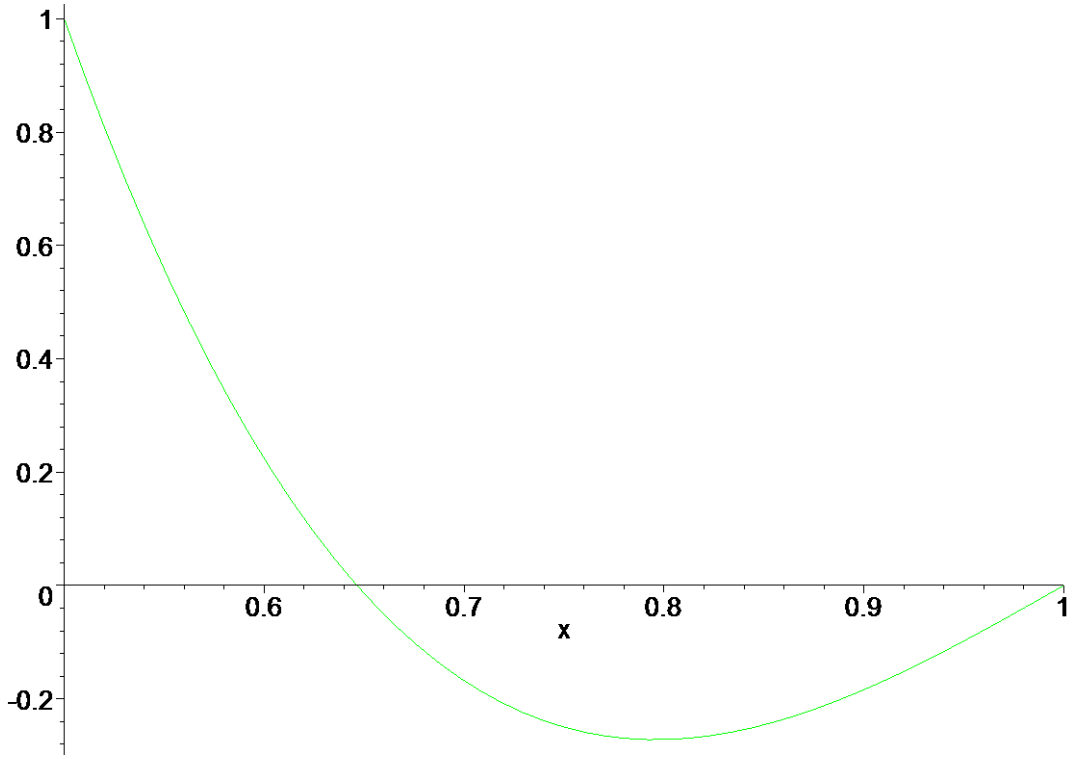
```
> plot([S3(x)], x=0.0..0.5, color=[blue], style=[line]);
```



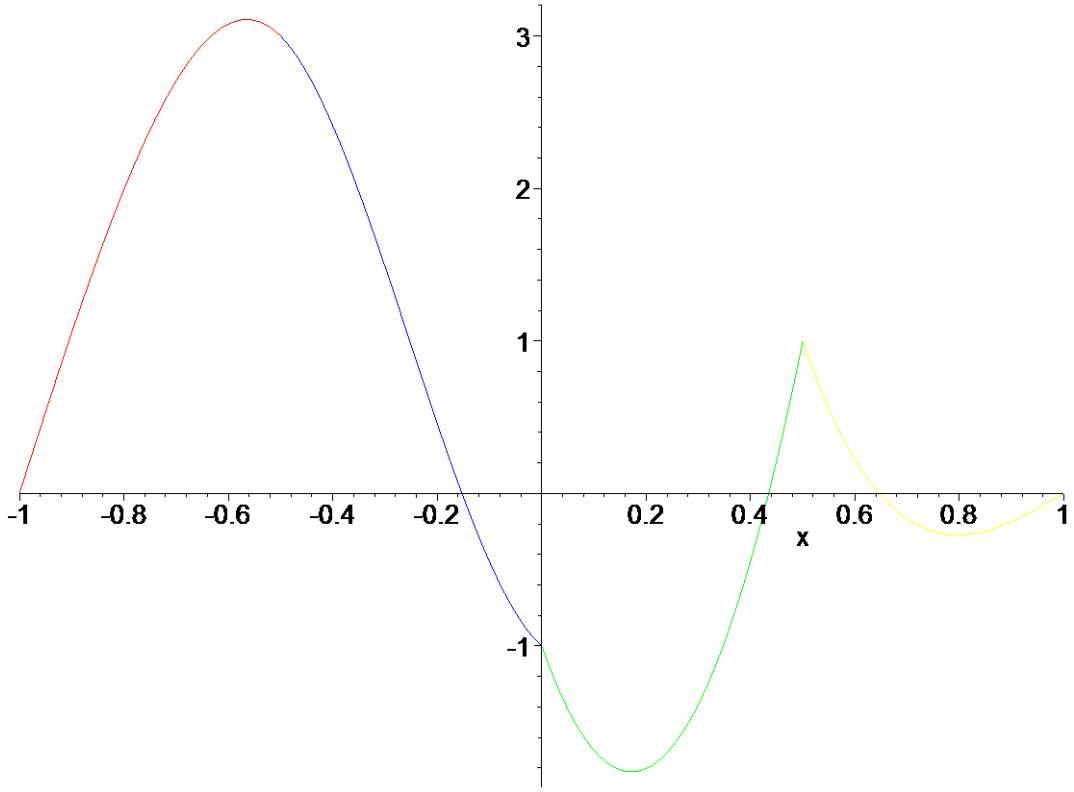

```
>
> S4:=y3+Y3[1]*(x-x3)+Y3[2]/2*(x-x3)^2+1/(6*h4)*(Y4[2]-Y3[2])*(x-x
  3)^3;
```

$$S4 := 6.000000000 - 10.00000000\,x + 24\,(x - .5)^2 - 16.00000000\,(x - .5)^3$$

```
> plot([S4(x)], x=0.5..1.0, color=[green], style=[line]);
```




```
> gr10:=plot(S1, x=-1.0..-0.5, color=[red]):
> gr20:=plot(S2, x=-0.5..0.0, color=[blue]):
> gr30:=plot(S3, x=0.0....0.5, color=[green]):
> gr40:=plot(S4, x=0.5....1.0, color=[yellow]):
> display(gr10,gr20,gr30,gr40);
```




```
>
```