POLINÔMIOS DE LEGENDRE: METODO DOS MÍNIMOS QUADRADOS

```
> restart:
> with(plots):
> with(linalg):
Warning, new definition for norm
Warning, new definition for trace
> f:= x-> exp(x);
```

$$f := \exp$$

```
> f(x):=exp(x);
```

$$f(x) := \mathbf{e}^{x}$$

```
> P0:= x->1;
```

$$P0 := 1$$

```
> P1:=x-> x;
```

$$P1 := x \to x$$

```
> P2:=x-> 3/2*x^2-1/2;
```

$$P2 := x \to \frac{3}{2}x^2 - \frac{1}{2}$$

```
> P3:=x-> 5/2*x^3-3/2*x;
```

$$P3 := x \to \frac{5}{2}x^3 - \frac{3}{2}x$$

```
> PL0(x):=1;
```

$$PL0(x) := 1$$

```
> PL1(x):= x;
```

$$PL1(x) := x$$

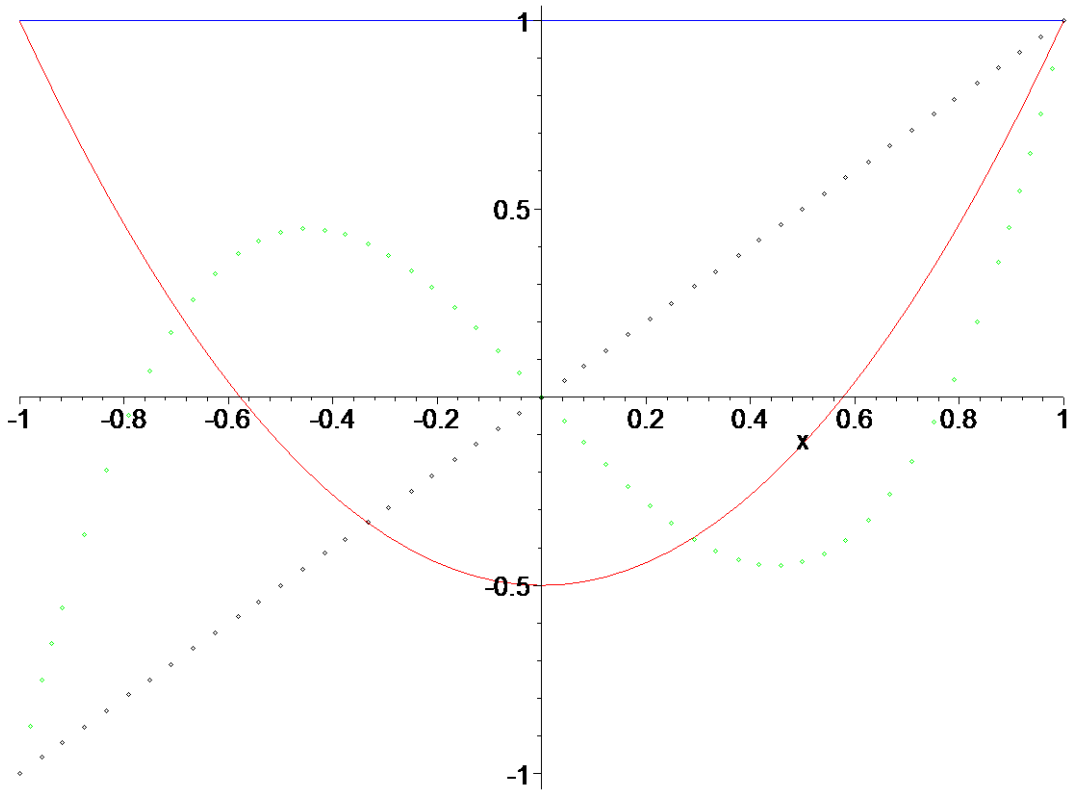
```
> PL2(x):=3/2*x^2-1/2;
```

$$PL2(x) := \frac{3}{2}x^2 - \frac{1}{2}$$

```
> PL3(x):= 5/2*x^3-3/2*x;
```

$$PL3(x) := \frac{5}{2}x^3 - \frac{3}{2}x$$

```
> L0:=plot([PL0(x)], x=-1.0..1.0, color=[blue], style=[line]):
> L1:=plot ([PL1(x)], x=-1.0..1.0, color=[black], style=[point]):
> L2:=plot ([PL2(x)], x=-1.0..1.0, color=[red], style=[line]):
> L3:=plot ([PL3(x)], x=-1.0..1.0, color=[green], style=[point]):
> display(L0,L1,L2,L3);
```



CALCULANDO OS POLINÔMIOS DE FORMA A SER O POLINÔMIO QUE AJUSTA A CURVA PELO MÉTODO DOS QUADRADOS MÍNIMOS.

POLINÔMIOS ORTOGONAIS: CASO CONTÍNUO

```
> int0:=simplify(int(P0*P0, x=-1..1));
```

$$int0 := 2$$

```
> int1:=simplify(int(P1(x)*P1(x), x=-1..1));
```

$$int1 := \frac{2}{3}$$

```
> int2:=simplify(int(P2(x)*P2(x), x=-1..1));
```

$$int2 := \frac{2}{5}$$

```
> int3:=simplify(int(P3(x)*P3(x), x=-1..1));
```

$$int3 := \frac{2}{7}$$

```
> b0:=simplify(int(P0(x)*f(x), x=-1..1));
```

$$b0 := -(-1 + \mathbf{e}^{(-2)})\,\mathbf{e}$$

```
> b1:=simplify(int(P1(x)*f(x), x=-1..1));
```

$$b1 := 2\,\mathbf{e}^{(-1)}$$

```
> b2:=simplify(int(P2(x)*f(x), x=-1..1));
```

$$b2 := -(-1 + 7\,\mathbf{e}^{(-2)})\,\mathbf{e}$$

```
> b3:=simplify(int(P3(x)*f(x), x=-1..1));
```

$$b3 := (-5 + 37\,\mathbf{e}^{(-2)})\,\mathbf{e}$$

```
> a0:=evalf(b0/int0);
```

$$a0 := 1.175201194$$

```
> a1:=evalf(b1/int1);
```

$$a1 := 1.103638324$$

```
> a2:=evalf(b2/int2);
```

$$a2 := .3578143523$$

```
> a3:=evalf(b3/int3);
```

$$a3 := .07045561698$$

```
> PL:=x-> a0*P0(x)+a1*P1(x)+a2*P2(x)+a3*P3(x);
```

$$PL := x \to a0\ \mathrm{P0}(x) + a1\ \mathrm{P1}(x) + a2\ \mathrm{P2}(x) + a3\ \mathrm{P3}(x)$$

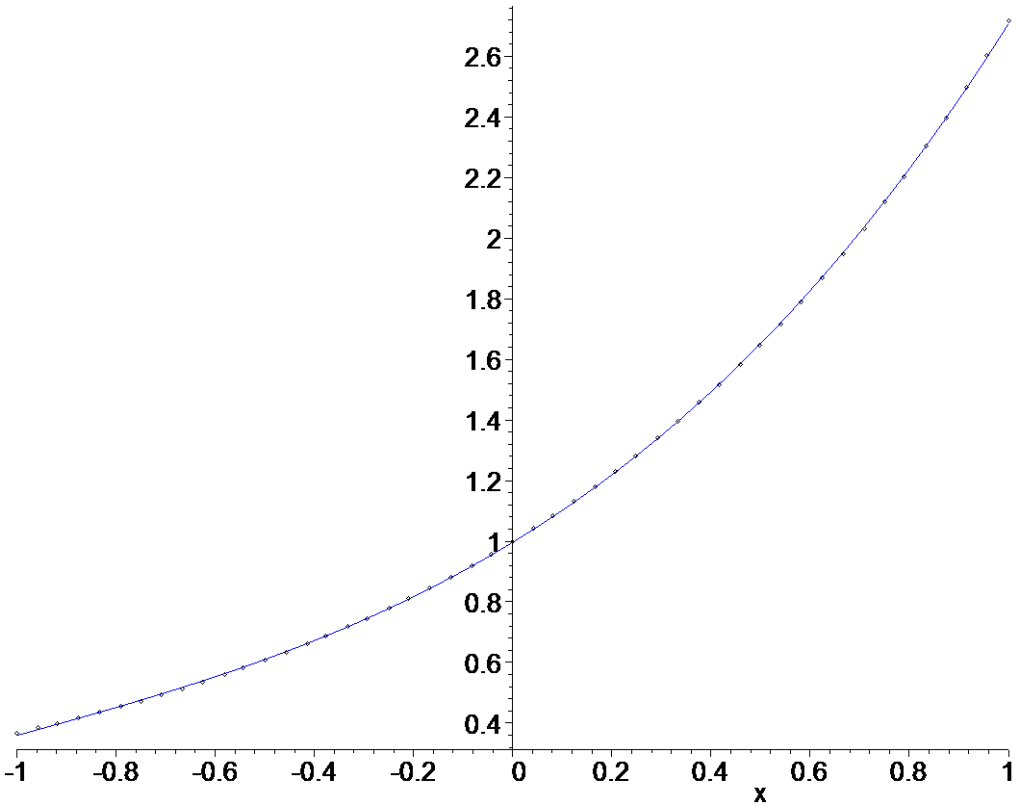
```
> f(x):=exp(x);
```

$$\mathrm{f}(x) := \mathbf{e}^{x}$$

```
> PL(x):=a0*P0(x)+a1*P1(x)+a2*P2(x)+a3*P3(x);
```

$$\mathrm{PL}(x) := .9962940178 + .9979548985\,x + .5367215285\,x^2 + .1761390425\,x^3$$

```
> z14:=plot([PL(x)], x=-1.0..1.0, color=[blue], style=[line]):
> z13:=plot ([f(x)], x=-1.0..1.0, color=[black], style=[point]):
> display(z13,z14);
```



POLINÔMIOS ORTOGONAIS: CASO DISCRETO

Ajuste de curvas por polinomios de grau 1 e 2.

Recorrência: P_{i+1}=(x-alpha{i+1}P_{i}+beta_i P_{i-1}

POLINÔMIOS DE GRAU 1

```
> x[1]:=-1.0:;
> x[2]:=-0.8:
> x[3]:=-0.6:
> x[4]:=-0.4:
> x[5]:=-0.2:
> x[6]:=-0.0:
> x[7]:=0.2:
> x[8]:=0.4:
> x[9]:=0.6:;
> x[10]:=0.8:
> x[11]:=1.0:
>
> N:=11:
> f(x):=exp(x);
```

$$f(x) := \mathbf{e}^{x}$$

```
> f(x[1]):;
> f(x[2]):;
> f(x[3]):;
> f(x[4]):;
> f(x[5]):;
> f(x[6]):;
> f(x[7]):;
> f(x[8]):;
> f(x[9]):;
> f(x[10]):;
> f(x[11]):;
> P_0:=x->1;
```

$$P\_0 := 1$$

```
> ALPHA1:=0;
```

$$ALPHA1 := 0$$

```
> ALPHA1:=ALPHA1+(1/N)*(x[1]+x[2]+x[3]+x[4]+x[5]+x[6]+x[7]+x[8]+x[
  9]+x[10]+x[11]);
```

$$ALPHA1 := 0$$

```
> P_1:=x->x-ALPHA1;
```

$$P\_1 := x \rightarrow x - ALPHA1$$

```
> a11:=0:
> a22:=0:
> b1:=0:
> b2:=0:
> for i from 1 to N do
> a11:=a11 +1;
> a22:=a22+(P_1(x[i]))^2;
> b1:=b1+f(x[i]):
> b2:=b2+f(x[i])*P_1(x[i]):
> od:;
```

```
> a_0:=b1/a11;
```

$$a\_0 := 1.212203623$$

```
> a_1:=b2/a22;
```

$$a\_1 := 1.123748300$$

```
> Q1(x):=a_0+a_1*P_1(x);
```

$$Q1(x) := 1.212203623 + 1.123748300\,x$$

```
> L:= vector([f(x[1]),f(x[2]), f(x[3]),
  f(x[4]),f(x[5]),f(x[6]),f(x[7]),f(x[8]),f(x[9]),f(x[10]),f(x[11]
  )]);
```

$L := [.3678794412, .4493289641, .5488116361, .6703200460, .8187307531, 1, 1.221402758, 1.491824698, 1.822118800, 2.225540928, 2.718281828]$

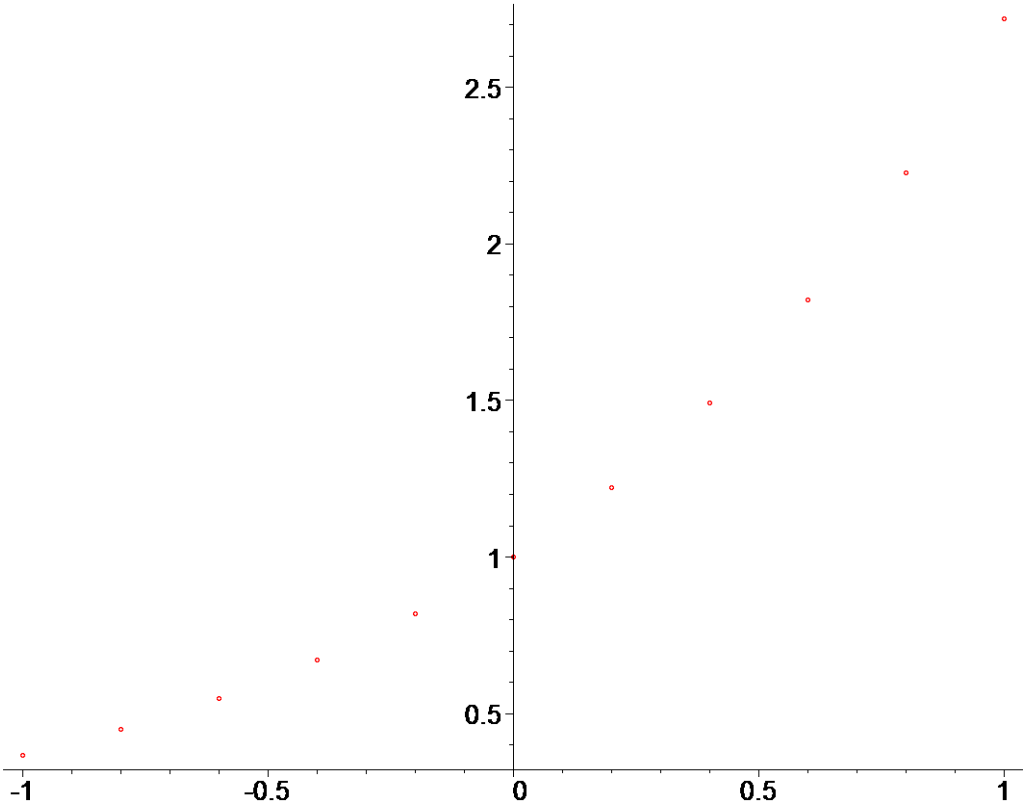
```
> X:=vector([x[1],x[2],x[3],x[4],x[5],x[6],x[7],x[8],x[9],x[10],x[
  11]]);
```

$X := [-1.0, -.8, -.6, -.4, -.2, 0, .2, .4, .6, .8, 1.0]$

```
> ZZ1:= [[ X[j], L[j]] $j=1..11];
```

$ZZ1 := [[-1.0, .3678794412], [-.8, .4493289641], [-.6, .5488116361], [-.4, .6703200460], [-.2, .8187307531], [0, 1], [.2, 1.221402758], [.4, 1.491824698], [.6, 1.822118800], [.8, 2.225540928], [1.0, 2.718281828]]$

```
> plot(ZZ1,style=point,symbol=circle);
```



POLINÔMIO DE GRAU 2

USANDO POLINÔMIOS ORTOGONAIS

Recorrência: P_{i+1}=(x-alpha{i+1}P_{i}+beta_i P_{i-1}

```
> BETA0:=0:;
```

```
> gama0:=0:;
> gama1:=0:;
> Q1 := x ->a_0+a_1*P_1(x);
```

$$Q1 := x \to a\_0 + a\_1\ \mathrm{P\_1}(x)$$

```
> for i from 1 to N do
> BETA0:=BETA0+x[i]*(P_1(x[i]));
> gama0:=gama0+(x[i]*(P_1(x[i])^2));
> gama1:=gama1+(P_1(x[i])^2):
> od:;
> BETA1:=(1/N)*BETA0;
```

$$BETA1 := .4000000000$$

```
> ALPHA2:=gama0/gama1;
```

$$ALPHA2 := 0$$

```
> P_2:=x ->(x-ALPHA2)*P_1(x)-BETA1*P_0(x);
```

$$P\_2 := x \to (x - ALPHA2)\ \mathrm{P\_1}(x) - BETA1\ \mathrm{P\_0}(x)$$

```
> P_2(x):=(x-ALPHA2)*P_1(x)-BETA1*P_0(x);
```

$$\mathrm{P\_2}(x) := x^2 - .4000000000$$

```
> a33:=0:
> b3:=0:
> for i from 1 to N do
> a33:=a33+(P_2(x[i]))^2:
> b3:=b3+f(x[i])*P_2(x[i]):
> od:;
> a_2:=b3/a33;
```

$$a\_2 := .5430255796$$

```
> Q_2:=x->a_0+a_1*P_1(x)+a_2*P_2(x);
```

$$Q\_2 := x \to a\_0 + a\_1\ \mathrm{P\_1}(x) + a\_2\ \mathrm{P\_2}(x)$$

```
> Q_2(x):=a_0+a_1*P_1(x)+a_2*P_2(x);
```

$$\mathrm{Q\_2}(x) := .9949933912 + 1.123748300\ x + .5430255796\ x^2$$

```
>
```

POLINÔMIO DE GRAU 3

```
> P_0:= x->1;
```

$$P\_0 := 1$$

```
> P_1:=x-> x-ALPHA1;
```

$$P\_1 := x \to x - ALPHA1$$

```
> P_2:=x ->(x-ALPHA2)*P_1(x)-BETA1*P_0(x);
```

$$P\_2 := x \to (x - ALPHA2)\ \mathrm{P\_1}(x) - BETA1\ \mathrm{P\_0}(x)$$

```
> P_2(x):=(x-ALPHA2)*P_1(x)-BETA1*P_0(x);
```

$$\mathrm{P\_2}(x) := x^2 - .4000000000$$

```
> BETA2:=0;
> BETA3:=0;
> ALPHA3:=0;
> ALPHA4:=0;
```

```
> for i from 1 to N do
> BETA2:=BETA2+(x[i]*(P_2(x[i]))*(P_1(x[i])));
> BETA3:=BETA3+(P_1(x[i]))^2;
> ALPHA3:=ALPHA3+(x[i]*(P_2(x[i]))^2);
> ALPHA4:=ALPHA4+ (P_2(x[i])^2);
> od:;
> BETA4:=BETA2/BETA3;
```

$$BETA4 := .3120000000$$

```
> ALPHA5:=ALPHA3/ALPHA4;
```

$$ALPHA5 := 0$$

```
>
> P_3:=x-> (x-ALPHA5)*P_2(x)-BETA4*P_1(x);
```

$$P\_3 := x \to (x - ALPHA5)\,\mathrm{P\_2}(x) - BETA4\,\mathrm{P\_1}(x)$$

```
> P_3(x):=(x-ALPHA5)*P_2(x)-BETA4*P_1(x);
```

$$\mathrm{P\_3}(x) := x\,(x^2 - .4000000000) - .3120000000\,x$$

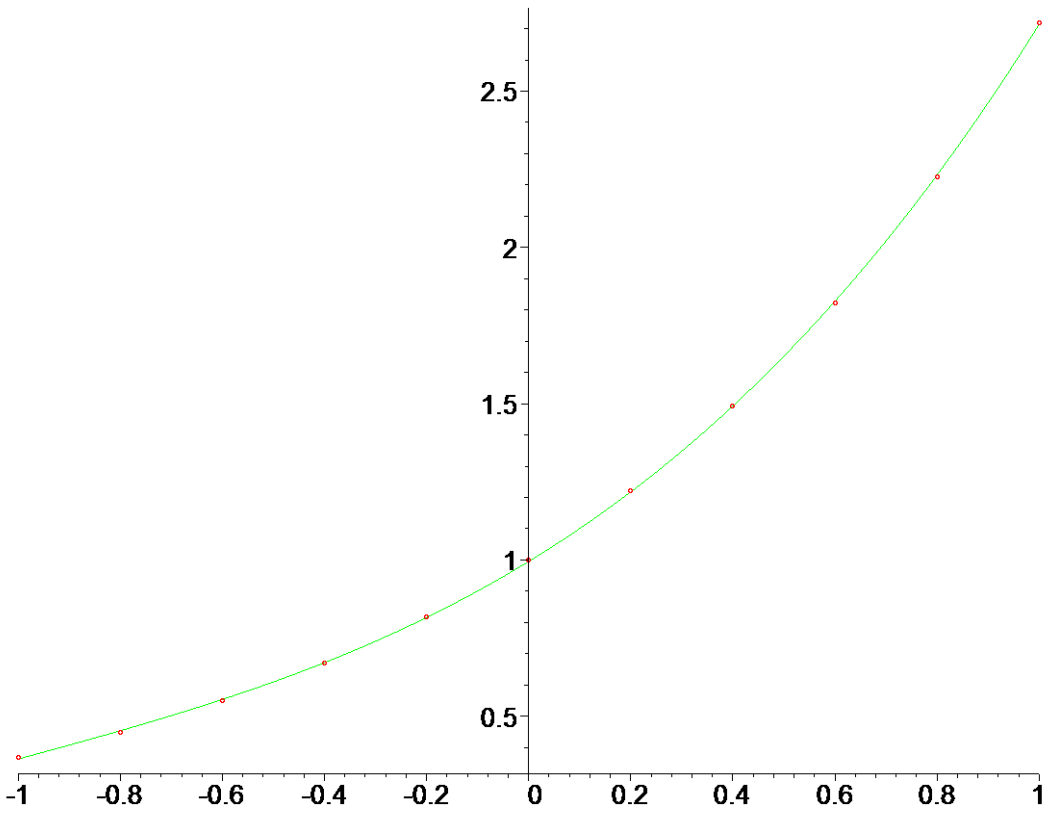
```
> a44:=0:
> b4:=0:
> for i from 1 to N do
> a44:=a44+(P_3(x[i])^2):
> b4:=b4+f(x[i])*(P_3(x[i]));
> od:;
> a3:=b4/a44;
```

$$a3 := .1774967739$$

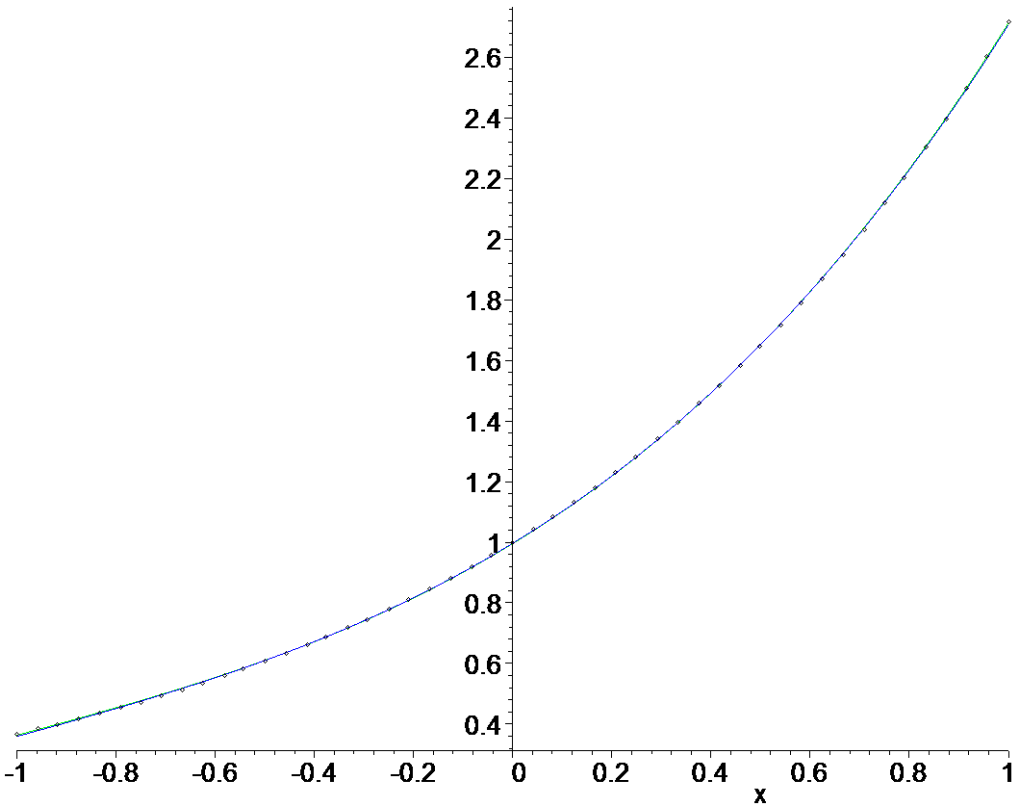
```
> Q3:=x-> a_0+a_1*P_1(x)+a_2*P_2(x)+a3*P_3(x):;
>
> Q_3(x):=a_0+a_1*P_1(x)+a_2*P_2(x)+a3*P_3(x);
```

$$\mathrm{Q\_3}(x) := .9949933912 + 1.068369307\,x + .5430255796\,x^2 + .1774967739\,x\,(x^2 - .4000000000)$$

```
> zb3:=plot([Q3(x)], x=-1.0..1.0, color=[green], style=[line]):
>
> display(plot([ZZ1],style=[point],symbol=circle),zb3);
```




```
> 
> display(z13,z14,zb3);
```




```
> Q_3(x) :=
  .9949933912+1.068369307*x+.5430255796*x^2+.1774967739*x*(x^2-.40
  00000000);
```

$$Q\_3(x) := $$

$$.9949933912 + 1.068369307\,x + .5430255796\,x^2 + .1774967739\,x\,(x^2 - .4000000000)$$

```
> PL(x) :=
  .9962940178+.9979548985*x+.5367215285*x^2+.1761390425*x^3;
```

$$PL(x) := .9962940178 + .9979548985\,x + .5367215285\,x^2 + .1761390425\,x^3$$

```
>
```

Observe que os polinomios de grau três de quadrados minimos para o caso continuo e caso discreto são praticamente iguais.