MÉTODOS NUMÉRICOS PARA RESOLUÇÃO DE SISTEMAS LINEARES (1,0,-1,0)
SOLUÇÃO EXATA DO SISTEMA X=(1,0,-1,0)

```
> with(linalg):
> restart:
> with(plots):
>
> a[11]*x_1+a[12]*x_2+a[13]*x_3+a[14]*x_4=b[1]:
> a[21]*x_1+a[22]*x_2+a[23]*x_3+a[24]*x_4=b[2]:
> a[31]*x_1+a[32]*x_2+a[33]*x_3+a[34]*x_4=b[3]:
> a[41]*x_1+a[42]*x_2+a[43]*x_3+a[44]*x_4=b[4]:
```

ATRIBUINDO VALORES

```
> a[11]:=6:;a[12]:=2:;a[13]:=1:;a[14]:=-2:;
> a[21]:=2:;a[22]:=8:;a[23]:=-1:;a[24]:=-1:;
> a[31]:=1:;a[32]:=-1:;a[33]:=5:;a[34]:=1:;
> a[41]:=-2:;a[42]:=-1:;a[43]:=1:;a[44]:=6:;
> b[1]:=5:;b[2]:=3:;b[3]:=-4:; b[4]:=-3:;
>
>
>
```

MÉTODOS DIRETOS

1 - MÉTODO DE ELIMINAÇÃO DE GAUSS COM PIVOTEAMENTO

```
> A:=matrix(4,4,[[a[11],a[12],a[13],a[14]],[a[21],a[22],a[23],a[24]]
,[a[31],a[32],a[33],a[34]],[a[41],a[42],a[43],a[44]]]):;
> b:=vector(4,[b[1],b[2],b[3],b[4]]):;
> A_0:=matrix(4,5,[[a[11],a[12],a[13],a[14],b[1]],[a[21],a[22],a[23]
,a[24],b[2]],[a[31],a[32],a[33],a[34],b[3]],[a[41],a[42],a[43],a[4
4],b[4]]]);
```

$$A\_0 := \begin{bmatrix} 6 & 2 & 1 & -2 & 5 \\ 2 & 8 & -1 & -1 & 3 \\ 1 & -1 & 5 & 1 & -4 \\ -2 & -1 & 1 & 6 & -3 \end{bmatrix}$$

```
>
```

A_0 é a matriz aumentada do sistema linear

```
> L_1:=vector(5,[a[11],a[12],a[13],a[14],b[1]]);
```

$$L\_1 := [6, 2, 1, -2, 5]$$

```
> L_2:=vector(5,[a[21],a[22],a[23],a[24],b[2]]);
```

$$L\_2 := [2, 8, -1, -1, 3]$$

```
> L_3:=vector(5,[a[31],a[32],a[33],a[34],b[3]]);
```

$$L\_3 := [1, -1, 5, 1, -4]$$

```
> L_4:=vector(5,[a[41],a[42],a[43],a[44],b[4]]);
```

$$L\_4 := [-2, -1, 1, 6, -3]$$

```
> m_21:=a[21]/a[11];
```

$$m\_21 := \frac{1}{3}$$

```
> m_31:=a[31]/a[11];
```

$$m\_31 := \frac{1}{6}$$

```
> m_41:=a[41]/a[11];
```

$$m\_41 := \frac{-1}{3}$$

```
> L_22:=evalm(L_2-m_21*L_1);
```

$$L\_22 := \left[0, \frac{22}{3}, \frac{-4}{3}, \frac{-1}{3}, \frac{4}{3}\right]$$

```
>
> L_32:=evalm(L_3-m_31*L_1);
```

$$L\_32 := \left[0, \frac{-4}{3}, \frac{29}{6}, \frac{4}{3}, \frac{-29}{6}\right]$$

```
> L_42:=evalm(L_4-m_41*L_1);
```

$$L\_42 := \left[0, \frac{-1}{3}, \frac{4}{3}, \frac{16}{3}, \frac{-4}{3}\right]$$

```
> m32:=(-4/3)/(22/3);
```

$$m32 := \frac{-2}{11}$$

```
> m42:=(-1/3)/(22/3);
```

$$m42 := \frac{-1}{22}$$

```
> L_33:=evalm(L_32-m32*L_22);
```

$$L\_33 := \left[0, 0, \frac{101}{22}, \frac{14}{11}, \frac{-101}{22}\right]$$

```
> L_43:=evalm(L_42-m42*L_22);
```

$$L\_43 := \left[0, 0, \frac{14}{11}, \frac{117}{22}, \frac{-14}{11}\right]$$

```
> m43:=(14/11)/(101/22);
```

$$m43 := \frac{28}{101}$$

```
> L_44:=evalm(L_43-m43*L_33);
```

$$L\_44 := \left[0, 0, 0, \frac{1003}{202}, 0\right]$$

```
> A_4:=matrix(4,5,[[6,2,1,-2,5],[0, 22/3, -4/3, -1/3, 4/3],[0, 0,
101/22, 14/11, -101/22],[0, 0, 0, 1003/202, 0]]);
```

```
>
```

$$A\_4 := \begin{bmatrix} 6 & 2 & 1 & -2 & 5 \\ 0 & \frac{22}{3} & \frac{-4}{3} & \frac{-1}{3} & \frac{4}{3} \\ 0 & 0 & \frac{101}{22} & \frac{14}{11} & \frac{-101}{22} \\ 0 & 0 & 0 & \frac{1003}{202} & 0 \end{bmatrix}$$

```
> x_4:=0/1003/202;
```

$$x\_4 := 0$$

```
> x_3:=(-101/22)/(101/22);
```

$$x\_3 := -1$$

```
> x_2:=4/3-(-4/3*x_3-1/3*x_4);
```

$$x\_2 := 0$$

```
> x_1:=(5-(2*x_2+1*x_3-2*x_4))/6;
```

$$x\_1 := 1$$

```
> x:=(x_1,x_2,x_3,x_4);
```

$$x := 1, 0, -1, 0$$

SOLUÇÃO DO SISTEMA PELO MEG X=(1,0,-1,0)

```
>
```

DECOMPOSIÇÃO LU

NOTE QUE A MATRIZ U=A_4(Matriz obtida pelo MEG)

L matriz triangular inferior com diagonal unitária, formada pelos multiplicadores do MEG a menos de trocas de linhas

```
>
> U:=
  LUdecomp(A,L='l',U='u',U1='u1',R='r',P='p',det='d',rank='ran');
```

$$U := \begin{bmatrix} 6 & 2 & 1 & -2 \\ 0 & \frac{22}{3} & \frac{-4}{3} & \frac{-1}{3} \\ 0 & 0 & \frac{101}{22} & \frac{14}{11} \\ 0 & 0 & 0 & \frac{1003}{202} \end{bmatrix}$$

```
> evalm(l);
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{3} & 1 & 0 & 0 \\ \frac{1}{6} & \frac{-2}{11} & 1 & 0 \\ \frac{-1}{3} & \frac{-1}{22} & \frac{28}{101} & 1 \end{bmatrix}$$

```
> AA:=multiply(l,U);
```

$$AA := \begin{bmatrix} 6 & 2 & 1 & -2 \\ 2 & 8 & -1 & -1 \\ 1 & -1 & 5 & 1 \\ -2 & -1 & 1 & 6 \end{bmatrix}$$

RESOLVENDO OS SISTEMAS: Ly=b e DEPOIS Ux=y
Ly=b (resolvendo por substituição obtemos)

```
>
> y:=linsolve(l,b);
```

$$y := \left[5, \frac{4}{3}, \frac{-101}{22}, 0 \right]$$

```
> x:=linsolve(U,y);
```

$$x := [1, 0, -1, 0]$$

MÉTODO DE CROUT ou L^T DL

Decomposição de uma matriz simétrica A= L^T DL
L: Matriz triangular inferior. e D matriz diagonal

```
> d[11]:=a[11];
```

$$d_{11} := 6$$

```
> c[21]:=a[21]/d[11]:; c[31]:=a[31]/a[11]:; c[41]:=a[41]/d[11]:;
> aux[21]:=c[21]*d[11]:;
> d[22]:=a[22]-c[21]*aux[21]:; c[32]:=(a[32]-c[31]*aux[21])/d[22]:;
  c[42]:=(a[42]-(c[41]*aux[21]))/d[22]:;
> aux[31]:=c[31]*d[11]:;aux[32]:=c[32]*d[22]:;
> d[33]:=a[33]-(c[31]*aux[31]+c[32]*aux[32]):;c[43]:=(a[43]-(c[41]*a
  ux[31]+c[42]*aux[32]))/d[33]:;
> aux[41]:=c[41]*d[11]:;aux[42]:=c[42]*d[22]:;aux[43]:=c[43]*d[33]:;
> d[44]:=a[44]-(c[41]*aux[41]+c[42]*aux[42]+c[43]*aux[43]):;
> LL:=matrix(4,4,[[1,0,0,0],[c[21],1,0,0],[c[31],c[32],1,0],[c[41],c
  [42],c[43],1]]);
```

$$LL := \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{3} & 1 & 0 & 0 \\ \frac{1}{6} & \frac{-2}{11} & 1 & 0 \\ \frac{-1}{3} & \frac{-1}{22} & \frac{28}{101} & 1 \end{bmatrix}$$

```
> DD:=matrix(4,4,[[d[11],0,0,0],[0,d[22],0,0],[0,0,d[33],0],[0,0,0,d[44]]]);
```

$$DD := \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & \frac{22}{3} & 0 & 0 \\ 0 & 0 & \frac{101}{22} & 0 \\ 0 & 0 & 0 & \frac{1003}{202} \end{bmatrix}$$

```
> UU:=multiply(DD,transpose(LL));
```

$$UU := \begin{bmatrix} 6 & 2 & 1 & -2 \\ 0 & \frac{22}{3} & \frac{-4}{3} & \frac{-1}{3} \\ 0 & 0 & \frac{101}{22} & \frac{14}{11} \\ 0 & 0 & 0 & \frac{1003}{202} \end{bmatrix}$$

Note que a multiplicação da matriz diagonal D, vezes a transposta da matriz L dá exatamente o valor a matriz triangular superior superior U,
obtida da decomposição LU., ou seja UU=D*L^t=U, logo tem-se que L*UU=L*D*L^t=A
RESOLVENDO O SISTEMA LINEAR: AX=b, L*D*L^t x=b,   Lz=b, Dy=z e (L^t) x = y: x é a solução.
NOTE QUE O det(A)=det(L*D*L^t)=det(D)=d11*d22*d33*d44, poi s det(LL)=1.

A multiplicação abaixo para obter é para simples conferencia que a decomposição da matriz A esta correta.

```
> AA:=multiply(LL,UU);
```

$$AA := \begin{bmatrix} 6 & 2 & 1 & -2 \\ 2 & 8 & -1 & -1 \\ 1 & -1 & 5 & 1 \\ -2 & -1 & 1 & 6 \end{bmatrix}$$

```
> zz:=linsolve(LL,b);
```

$$zz := \left[5, \frac{4}{3}, \frac{-101}{22}, 0 \right]$$

```
> yy:=linsolve(DD,zz);
```

$$yy := \left[\frac{5}{6}, \frac{2}{11}, -1, 0\right]$$

```
> xx:=linsolve(transpose(LL),yy);
```

$$xx := [1, 0, -1, 0]$$

MÉTODO DE CHOLESKY A=L*(transposta de L)
Matriz dos coeficientes A é simetrica e definida positiva.

```
>
>
L:=cholesky(A);
```

$$L := \begin{bmatrix} \sqrt{6} & 0 & 0 & 0 \\ \frac{1}{3}\sqrt{6} & \frac{1}{3}\sqrt{66} & 0 & 0 \\ \frac{1}{6}\sqrt{6} & -\frac{2}{33}\sqrt{66} & \frac{1}{22}\sqrt{2222} & 0 \\ -\frac{1}{3}\sqrt{6} & -\frac{1}{66}\sqrt{66} & \frac{14}{1111}\sqrt{2222} & \frac{1}{202}\sqrt{202606} \end{bmatrix}$$

Ly=b   e transposta(L)x=y

```
> AB:=multiply(L,transpose(L));
```

$$AB := \begin{bmatrix} 6 & 2 & 1 & -2 \\ 2 & 8 & -1 & -1 \\ 1 & -1 & 5 & 1 \\ -2 & -1 & 1 & 6 \end{bmatrix}$$

```
> y1:=multiply(inverse(L),b);
```

$$y1 := \left[\frac{5}{6}\sqrt{6}, \frac{2}{33}\sqrt{66}, -\frac{1}{22}\sqrt{2222}, 0\right]$$

```
> xx:=multiply(inverse(transpose(L)),y1);
```

$$xx := [1, 0, -1, 0]$$

```
>
```

MÉTODOS ITERATIVOS

1 - MÉTODO DE GAUSS-JACOBI

```
>
> 6*x_1+2*x_2+x_3-2*x_4=5:
> 2*x_1+8*x_2-x_3-x_4=3:
> x_1-x_2+5*x_3+x_4=-4:
> -2*x_1-x_2+1*x_3+6*x_4=-3:
> with(linalg):
>
> x1:=(5-(2*x_2+x_3-2*x_4))/6:
> x2:=(3-(2*x_1-x_3-x_4))/8:
```

```
> x3:=(-4-(x_1-x_2+x_4))/5:
> x4:=(-3-(-2*x_1-x_2+1*x_3))/6:
>
```

INICIANDO O PROCESSO ITERATIVO

```
> x1[1]:=5:
> x2[1]:=3:
> x3[1]:=-4:
> x4[1]:=-3:
> for i from 1 to 20 do
> x1[i+1]:=evalf((5-(2*x2[i]+x3[i]-2*x4[i]))/6,5):
> x2[i+1]:=evalf((3-(2*x1[i]-x3[i]-x4[i]))/8,5):
> x3[i+1]:=evalf((-4-(x1[i]-x2[i]+x4[i]))/5,5):
> x4[i+1]:=evalf((-3-(-2*x1[i]-x2[i]+x3[i]))/6,5):
> E[i]:=evalf(max(abs(x1[i+1]-x1[i]),abs(x2[i+1]-x2[i]),abs(x3[i+1]-
  x3[i]),abs(x4[i+1]-x4[i])),8):
> X[i+1]:=evalf(vector(4,[x1[i+1],x2[i+1],x3[i+1],x4[i+1]]),5);
> od;
>
```

$$x1_2 := -.50000$$

$$x2_2 := -1.7500$$

$$x3_2 := -.60000$$

$$x4_2 := 2.3333$$

$$E_1 := 5.50000$$

$$X_2 := [-.50000, -1.7500, -.60000, 2.3333]$$

$$x1_3 := 2.2945$$

$$x2_3 := .71666$$

$$x3_3 := -1.5167$$

$$x4_3 := -.85834$$

$$E_2 := 3.19164$$

$$X_3 := [2.2945, .71666, -1.5167, -.85834]$$

$$x1_4 := .56111$$

$$x2_4 := -.49551$$

$$x3_4 := -.94393$$

$$x4_4 := .63705$$

$$E_3 := 1.73339$$

$$X_4 := [.56111, -.49551, -.94393, .63705]$$

$x1_5 := 1.3682$

$x2_5 := .19636$

$x3_5 := -1.1387$

$x4_5 := -.23823$

$E_4 := .87528$

$X_5 := [1.3682, .19636, -1.1387, -.23823]$

$x1_6 := .87825$

$x2_6 := -.13917$

$x3_6 := -.98665$

$x4_6 := .17858$

$E_5 := .48995$

$X_6 := [.87825, -.13917, -.98665, .17858]$

$x1_7 := 1.1037$

$x2_7 := .054433$

$x3_7 := -1.0392$

$x4_7 := -.06601$

$E_6 := .24459$

$X_7 := [1.1037, .054433, -1.0392, -.06601]$

$x1_8 := .96639$

$x2_8 := -.039081$

$x3_8 := -.99660$

$x4_8 := .05017$

$E_7 := .13731$

$X_8 := [.96639, -.039081, -.99660, .05017]$

$x1_9 := 1.0292$

$x2_9 := .015091$

$x3_9 := -1.0111$

$x4_9 := -.01828$

$E_8 := .06845$

$X_9 := [1.0292, .015091, -1.0111, -.01828]$

$x1_{10} := .99073$

$x2_{10} := -.010975$

$x3_{10} := -.99914$

$x4_{10} := .01411$

$E_9 := .03847$

$X_{10} := [\,.99073, -.010975, -.99914, .01411\,]$

$x1_{11} := 1.0082$

$x2_{11} := .0041938$

$x3_{11} := -1.0031$

$x4_{11} := -.00507$

$E_{10} := .01918$

$X_{11} := [\,1.0082, .0041938, -1.0031, -.00507\,]$

$x1_{12} := .99742$

$x2_{12} := -.0030738$

$x3_{12} := -.99979$

$x4_{12} := .00395$

$E_{11} := .01078$

$X_{12} := [\,.99742, -.0030738, -.99979, .00395\,]$

$x1_{13} := 1.0023$

$x2_{13} := .0011638$

$x3_{13} := -1.0009$

$x4_{13} := -.00141$

$E_{12} := .00536$

$X_{13} := [\,1.0023, .0011638, -1.0009, -.00141\,]$

$x1_{14} := .99929$

$x2_{14} := -.00086625$

$x3_{14} := -1.0000$

$x4_{14} := .00111$

$E_{13} := .00301$

$X_{14} := [\,.99929, -.00086625, -1.0000, .00111\,]$

$x1_{15} := 1.0007$

$x2_{15} := .00031875$

$x3_{15} := -1.0002$

$x4_{15} := -.00037$

$$E_{14} := .00148$$

$$X_{15} := [1.0007, .00031875, -1.0002, -.00037]$$

$$x1_{16} := .99980$$

$$x2_{16} := -.00025625$$

$$x3_{16} := -.99993$$

$$x4_{16} := .00032$$

$$E_{15} := .00090$$

$$X_{16} := [.99980, -.00025625, -.99993, .00032]$$

$$x1_{17} := 1.0002$$

$$x2_{17} := .00010000$$

$$x3_{17} := -1.0001$$

$$x4_{17} := -.00011$$

$$E_{16} := .00043$$

$$X_{17} := [1.0002, .00010000, -1.0001, -.00011]$$

$$x1_{18} := .99994$$

$$x2_{18} := -.000073750$$

$$x3_{18} := -.99996$$

$$x4_{18} := .00010$$

$$E_{17} := .00026$$

$$X_{18} := [.99994, -.000073750, -.99996, .00010]$$

$$x1_{19} := 1.0000$$

$$x2_{19} := .000022500$$

$$x3_{19} := -1.0000$$

$$x4_{19} := -.00004$$

$$E_{18} := .00014$$

$$X_{19} := [1.0000, .000022500, -1.0000, -.00004]$$

$$x1_{20} := .99998$$

$$x2_{20} := -.50000\ 10^{-5}$$

$$x3_{20} := -.99999$$

$$x4_{20} := 0$$

$$E_{19} := .00004$$

$$X_{20} := [.99998, -.50000\ 10^{-5}, -.99999, 0]$$

$$x1_{21} := 1.0000$$

$$x2_{21} := 0$$

$$x3_{21} := -1.0000$$

$$x4_{21} := 0$$

$$E_{20} := .00002$$

$$X_{21} := [1.0000, 0, -1.0000, 0]$$

Foram necessários 21 iterações para obter a solução " aproximada"

2 - MÉTODO DE GAUSS-SEIDEL

```
>
> x1[1]:=5:
> x2[1]:=3:
> x3[1]:=-4:
> x4[1]:=-3:
>
> for i from 1 to 12 do
> x1[i+1]:=evalf((5-(2*x2[i]+x3[i]-2*x4[i]))/6,5):
> x2[i+1]:=evalf((3-(2*x1[i+1]-x3[i]-x4[i]))/8,5):
> x3[i+1]:=evalf((-4-(x1[i+1]-x2[i+1]+x4[i]))/5,5):
> x4[i+1]:=evalf((-3-(-2*x1[i+1]-x2[i+1]+x3[i+1]))/6,5):
> E[i]:=evalf(max(abs(x1[i+1]-x1[i]),abs(x2[i+1]-x2[i]),abs(x3[i+1]-
  x3[i]),abs(x4[i+1]-x4[i])),8):
>
> X[i+1]:=evalf(vector(4,[x1[i+1],x2[i+1],x3[i+1],x4[i+1]]),5);
> od;
>
```

$$x1_{2} := -.50000$$

$$x2_{2} := -.37500$$

$$x3_{2} := -.17500$$

$$x4_{2} := -.70000$$

$$E_{1} := 5.50000$$

$$X_{2} := [-.50000, -.37500, -.17500, -.70000]$$

$$x1_{3} := .75417$$

$$x2_{3} := .077090$$

$x3_3 := -.79541$

$x4_3 := -.10319$

$E_2 := 1.25417$

$X_3 := [.75417, .077090, -.79541, -.10319]$

$x1_4 := .90580$

$x2_4 := .036225$

$x3_4 := -.95328$

$x4_4 := -.03315$

$E_3 := .15787$

$X_4 := [.90580, .036225, -.95328, -.03315]$

$x1_5 := .96909$

$x2_5 := .0094262$

$x3_5 := -.98530$

$x4_5 := -.01118$

$E_4 := .06329$

$X_5 := [.96909, .0094262, -.98530, -.01118]$

$x1_6 := .99068$

$x2_6 := .0027725$

$x3_6 := -.99535$

$x4_6 := -.00342$

$E_5 := .02159$

$X_6 := [.99068, .0027725, -.99535, -.00342]$

$x1_7 := .99716$

$x2_7 := .00086250$

$x3_7 := -.99858$

$x4_7 := -.00104$

$E_6 := .00648$

$X_7 := [.99716, .00086250, -.99858, -.00104]$

$x1_8 := .99912$

$x2_8 := .00027000$

$x3_8 := -.99956$

$x4_8 := -.00033$

$$E_{7} := .00196$$

$$X_{8} := [.99912, .00027000, -.99956, -.00033]$$

$$x1_{9} := .99972$$

$$x2_{9} := .000078750$$

$$x3_{9} := -.99985$$

$$x4_{9} := -.00011$$

$$E_{8} := .00060$$

$$X_{9} := [.99972, .000078750, -.99985, -.00011]$$

$$x1_{10} := .99990$$

$$x2_{10} := .000026250$$

$$x3_{10} := -.99995$$

$$x4_{10} := -.00004$$

$$E_{9} := .00018$$

$$X_{10} := [.99990, .000026250, -.99995, -.00004]$$

$$x1_{11} := .99997$$

$$x2_{11} := .000015000$$

$$x3_{11} := -.99998$$

$$x4_{11} := -.00002$$

$$E_{10} := .00007$$

$$X_{11} := [.99997, .000015000, -.99998, -.00002]$$

$$x1_{12} := .99998$$

$$x2_{12} := -.25000\ 10^{-5}$$

$$x3_{12} := -1.0000$$

$$x4_{12} := 0$$

$$E_{11} := .00002$$

$$X_{12} := [.99998, -.25000\ 10^{-5}, -1.0000, 0]$$

$$x1_{13} := 1.0000$$

$$x2_{13} := 0$$

$$x3_{13} := -1.0000$$

$$x4_{13} := 0$$

$$E_{12} := .00002$$

$X_{13} := [1.0000, 0, -1.0000, 0]$

```
>
```

Foram necessários 13 iterações para obter a solução aproximada, com 5 casas decimais
COMPARANDO OS DOIS MÉTODOS(GAUSS-JACOBI x GAUSS-SEIDEL, PODEMOS VER QUE O SEGUNDO CONVERGE COM 13 ITERAÇÕES E O PRIMEIRO COM 21 ITERAÇÕES, CONSIDERANDO APENAS 5 CASAS DECIMAIS

```
>
```

3 - MÉTODO DOS GRADIENTES

```
> with(linalg):
> x1[0]:=5:
> x2[0]:=3:
> x3[0]:=-4:
> x4[0]:=-3:
>
> b:=array(1..4,[5,3,-4,-3]);
```

$b := [5, 3, -4, -3]$

```
> A:=matrix(4,4,[[6,2,1,-2],[2,8,-1,-1],[1,-1,5,1],[-2,-1,1,6]]):
> X[0]:= array(1..4,[x1[0],x2[0],x3[0],x4[0]]);
```

$X_0 := [5, 3, -4, -3]$

```
>
> for i from 1 to 21 do
> r[i-1]:= evalf(evalm(b-multiply(A,X[i-1])),5);
> erro[i]:=evalf(evalm(max(abs(r[i-1]))),5);
> alpha[i-1]:=evalf(evalm(multiply(r[i-1],r[i-1])/(multiply(r[i-1],m
  ultiply(A,r[i-1])))),5);
> X[i]:=evalf(evalm(X[i-1]+alpha[i-1]*r[i-1]),5);
> od;
```

$r_0 := [-33., -38., 17., 32.]$

$erro_1 := [33., 38., 17., 32.]$

$\alpha_0 := .099624$

$X_1 := [1.7124, -.7857, -2.3064, .1880]$

$r_1 := [-1.0202, 3.7424, 4.8459, .8175]$

$erro_2 := [1.0202, 3.7424, 4.8459, .8175]$

$\alpha_1 := .21369$

$X_2 := [1.4944, .01401, -1.2709, .36269]$

$r_2 := [-1.9981, -1.0091, .5114, -.9024]$

$erro_3 := [1.9981, 1.0091, .5114, .9024]$

$$\alpha_2 := .17197$$

$$X_3 := [1.1508, -.15952, -1.1830, .20750]$$

$$r_3 := [.0122, .9991, .3972, -.9199]$$

$$erro_4 := [.0122, .9991, .3972, .9199]$$

$$\alpha_3 := .14032$$

$$X_4 := [1.1525, -.01933, -1.1273, .07842]$$

$$r_4 := [-.5922, -.1993, .3863, -.0575]$$

$$erro_5 := [.5922, .1993, .3863, .0575]$$

$$\alpha_4 := .17220$$

$$X_5 := [1.0505, -.053649, -1.0608, .068519]$$

$$r_5 := [.0021, .3359, .1314, -.3029]$$

$$erro_6 := [.0021, .3359, .1314, .3029]$$

$$\alpha_5 := .14032$$

$$X_6 := [1.0508, -.006516, -1.0424, .026016]$$

$$r_6 := [-.1974, -.0659, .1287, -.0186]$$

$$erro_7 := [.1974, .0659, .1287, .0186]$$

$$\alpha_6 := .17220$$

$$X_7 := [1.0168, -.017864, -1.0202, .022813]$$

$$r_7 := [.0007, .1119, .0435, -.1010]$$

$$erro_8 := [.0007, .1119, .0435, .1010]$$

$$\alpha_7 := .14019$$

$$X_8 := [1.0169, -.002177, -1.0141, .008654]$$

$$r_8 := [-.0656, -.0218, .0427, -.0062]$$

$$erro_9 := [.0656, .0218, .0427, .0062]$$

$$\alpha_8 := .17237$$

$$X_9 := [1.0056, -.0059347, -1.0067, .0075853]$$

$$r_9 := [.0002, .0372, .0144, -.0335]$$

$$erro_{10} := [.0002, .0372, .0144, .0335]$$

$$\alpha_9 := .14016$$

$$X_{10} := [1.0056, -.0007207, -1.0047, .0028899]$$

$$r_{10} := [-.0217, -.0072, .0143, -.0021]$$

$$erro_{11} := [.0217, .0072, .0143, .0021]$$

$$\alpha_{10} := .17310$$

$$X_{11} := [1.0018, -.0019670, -1.0022, .0025264]$$

$$r_{11} := [.0004, .0124, .0047, -.0114]$$

$$erro_{12} := [.0004, .0124, .0047, .0114]$$

$$\alpha_{11} := .13769$$

$$X_{12} := [1.0019, -.0002596, -1.0016, .0009567]$$

$$r_{12} := [-.0074, -.0023, .0048, -.0006]$$

$$erro_{13} := [.0074, .0023, .0048, .0006]$$

$$\alpha_{12} := .17346$$

$$X_{13} := [1.0006, -.00065856, -1.0008, .00085262]$$

$$r_{13} := [.0002, .0042, .0018, -.0038]$$

$$erro_{14} := [.0002, .0042, .0018, .0038]$$

$$\alpha_{13} := .13897$$

$$X_{14} := [1.0006, -.00007489, -1.0005, .00032453]$$

$$r_{14} := [-.0024, -.0008, .0015, -.0003]$$

$$erro_{15} := [.0024, .0008, .0015, .0003]$$

$$\alpha_{14} := .17449$$

$$X_{15} := [1.0002, -.00021448, -1.0002, .00027218]$$

$$r_{15} := [-.0001, .0014, .0003, -.0012]$$

$$erro_{16} := [.0001, .0014, .0003, .0012]$$

$$\alpha_{15} := .13709$$

$$X_{16} := [1.0002, -.00002255, -1.0002, .00010767]$$

$$r_{16} := [-.0008, -.0003, .0007, 0]$$

$$erro_{17} := [.0008, .0003, .0007, 0]$$

$$\alpha_{16} := .16781$$

$$X_{17} := [1.0001, -.000072893, -1.0001, .00010767]$$

$$r_{17} := [-.0002, .0004, .0002, -.0004]$$

$$erro_{18} := [.0002, .0004, .0002, .0004]$$

$$\alpha_{17} := .20408$$

$$X_{18} := [1.0001, .8739\ 10^{-5}, -1.0001, .000026038]$$

$$r_{18} := [-.0004, -.0004, .0004, .0001]$$

$$erro_{19} := [.0004, .0004, .0004, .0001]$$

$$\alpha_{18} := .12069$$

$$X_{19} := [1.0001, -.000039537, -1.0001, .000038107]$$

$$r_{19} := [-.0003, 0, .0004, .0001]$$

$$erro_{20} := [.0003, 0, .0004, .0001]$$

$$\alpha_{19} := .19118$$

$$X_{20} := [1.0000, -.000039537, -1.0000, .000057225]$$

$$r_{20} := [.0002, .0004, -.0001, -.0003]$$

$$erro_{21} := [.0002, .0004, .0001, .0003]$$

$$\alpha_{20} := .099668$$

$$X_{21} := [1.0000, .330\ 10^{-6}, -1.0000, .000027325]$$

Para obter a solução numérica aproximada foram necessárias 21 iterações, considerando 5 casas decimais

```
>
>
```

```
>
```

4 - MÉTODO DOS GRADIENTES CONJUGADOS

```
> with(linalg):
> x1[0]:=5:
> x2[0]:=3:
> x3[0]:=-4:
> x4[0]:=-3:
>
> b:=array(1..4,[5,3,-4,-3]);
```

$$b := [5, 3, -4, -3]$$

```
> A:=matrix(4,4,[[6,2,1,-2],[2,8,-1,-1],[1,-1,5,1],[-2,-1,1,6]]):
> X[0]:= array(1..4,[x1[0],x2[0],x3[0],x4[0]]);
```

$$X_0 := [5, 3, -4, -3]$$

```
> r[0]:= evalf(evalm(b-multiply(A,X[0])),5);
```

$$r_0 := [-33., -38., 17., 32.]$$

```
> P[0]:=evalm(r[0]);
```

$$P_0 := [-33., -38., 17., 32.]$$

```
>
> for i from 1 to 4 do
> alpha[i-1]:=evalf(evalm(multiply(r[i-1],P[i-1])/(multiply(P[i-1],m
  ultiply(A,P[i-1])))),5);
```

```
>
> X[i]:= evalf(evalm(X[i-1]+alpha[i-1]*P[i-1]),5);
> r[i]:= evalf(evalm(r[i-1]-alpha[i-1]*multiply(A,P[i-1])),5);
> erro[i]:=evalf(evalm(max(abs(r[i]))),5);
>
> beta[i]:=evalf(evalm(multiply(r[i],multiply(A,P[i-1]))/multiply(P[
  i-1],multiply(A,P[i-1]))),5);
> P[i]:=evalf(evalm(r[i]-beta[i]*P[i-1]),5);
> od;
>
>
```

$$\alpha_0 := .099624$$

$$X_1 := [1.7124, -.7857, -2.3064, .1880]$$

$$r_1 := [-1.021, 3.742, 4.846, .818]$$

$$erro_1 := [1.021, 3.742, 4.846, .818]$$

$$\beta_1 := -.010179$$

$$P_1 := [-1.3569, 3.3552, 5.0190, 1.1437]$$

$$\alpha_1 := .21848$$

$$X_2 := [1.4159, -.05266, -1.2098, .43788]$$

$$r_2 := [-1.3052, -.1830, .1428, -1.6377]$$

$$erro_2 := [1.3052, .1830, .1428, 1.6377]$$

$$\beta_2 := -.11325$$

$$P_2 := [-1.4589, .19698, .71120, -1.5082]$$

$$\alpha_2 := .28828$$

$$X_3 := [.99533, .004125, -1.0048, .00310]$$

$$r_3 := [.0301, -.02589, .02982, -.0184]$$

$$erro_3 := [.0301, .02589, .02982, .0184]$$

$$\beta_3 := -.00066619$$

$$P_3 := [.029128, -.025759, .030294, -.019405]$$

$$\alpha_3 := .15882$$

$$X_4 := [.99996, .0000340, -.99999, .0000181]$$

$$r_4 := [-.000449, -.000685, .000127, .000441]$$

$$erro_4 := [.000449, .000685, .000127, .000441]$$

$$\beta_4 := -.00035266$$

$$P_4 := [-.00043873, -.00\text{[illegible]}3, .00013768, .00043416]$$

>

NOTE QUE O MÉTODO DOS GRADIENTES CONJUGADOS CONVERGE MUITO MAIS RÁPIDO QUE O MÉTODO DOS GRADIENTES. TAMBÉM PODEMOS NOTAR QUE O MÉTODOS DOS GRADIENTES CONJUGADOS É UM POUCO MAIS RÁPIDO QUE O MÉTODO DE GAUSS-SEIDEL

>

>

EXEMPLOS DE SPLINES LINEAR E CÚBICA

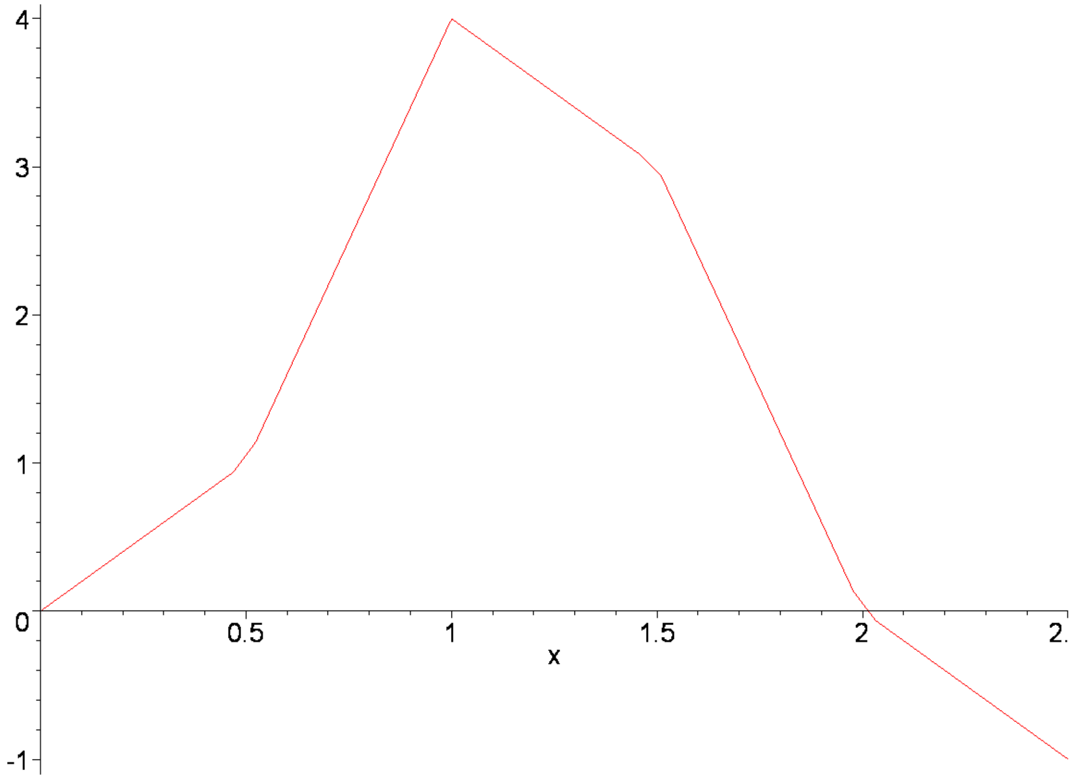
>

```
> readlib(spline):
  LI(x):=
  evalf((spline([0,0.5,1.0,1.5,2.0,2.5],[0,1,4,3,0,-1],x,linear)),3)
  ;
> SPL(x):=evalf((spline([0,0.5,1.0,1.5,2.0,2.5],[0,1,4,3,0,-1],x,cub
  ic)),5);
```

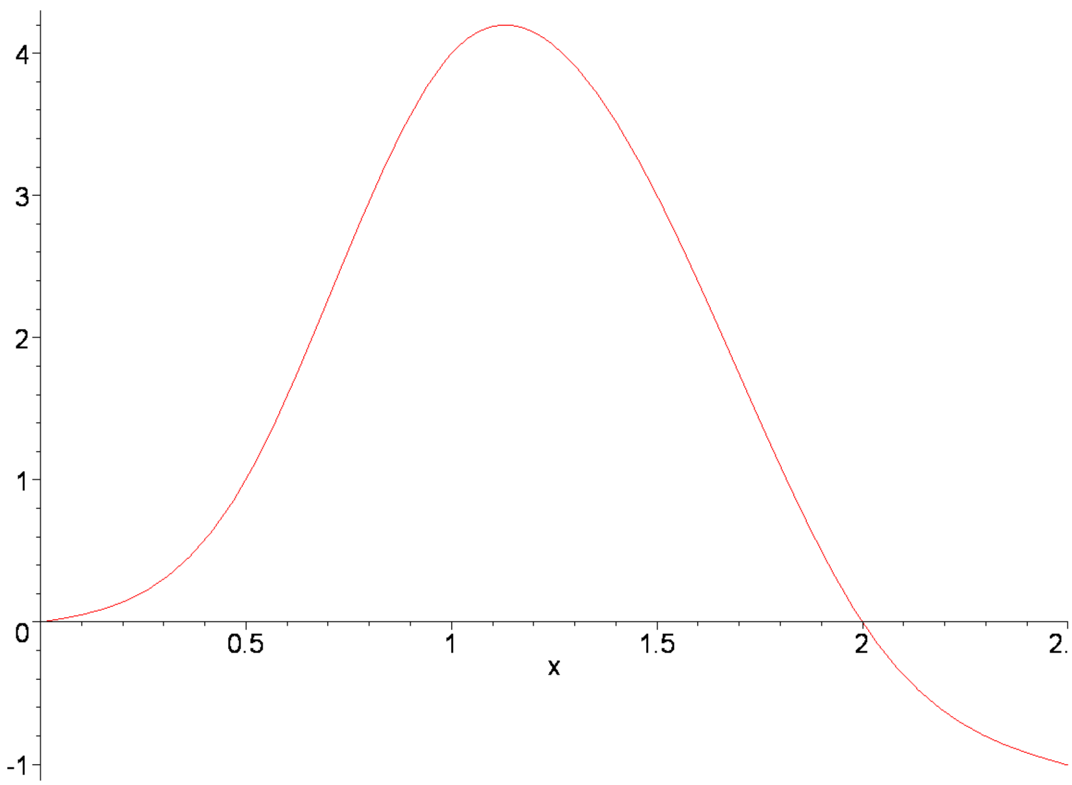
$$\mathrm{LI}(x) := \begin{cases} 2.00\,x & x < .5 \\ -2.00 + 6.00\,x & x < 1.0 \\ 6.00 - 2.00\,x & x < 1.5 \\ 12.0 - 6.00\,x & x < 2.0 \\ 4.00 - 2.00\,x & \mathit{otherwise} \end{cases}$$

$$\mathrm{SPL}(x) := \begin{cases} .450\,x + 6.201\,x^3 & x < .5 \\ 2.6505 - 15.456\,x + 31.812\,x^2 - 15.007\,x^3 & x < 1.0 \\ -18.169 + 47.004\,x - 30.648\,x^2 + 5.8135\,x^3 & x < 1.5 \\ -24.664 + 59.994\,x - 39.308\,x^2 + 7.7381\,x^3 & x < 2.0 \\ 75.212 - 89.818\,x + 35.598\,x^2 - 4.7463\,x^3 & \mathit{otherwise} \end{cases}$$

```
> plot(LI(x), x=0..2.5);
```




```
> plot(SPL(x), x=0..2.5);
```




```
>
```