METODO DOS MÍNIMOS QUADRADOS

APROXIMAÇÃO POR EXCPONENCIAL :EXP(X)

```
> restart:
> with(plots):
> with(linalg):
Warning, new definition for norm
Warning, new definition for trace
> x1[0]:=-1:
> x2[0]:=-0.7:
> x3[0]:=-0.4:
> x4[0]:=-0.1:
> x5[0]:=0.2:
> x6[0]:=0.5:
> x7[0]:=0.8:
> x8[0]:=1.0:
```

COLOCANDO OS PESOS W_i UNIFORMES

```
> w1[0]:=1:;
> w2[0]:=1:;
> w3[0]:=1:;
> w4[0]:=1:;
> w5[0]:=1:;
> w6[0]:=1:;
> w7[0]:=1:;
> w8[0]:=1:;
```

VALOR DA FUNÇÃO NOS PONTOS

```
> f1[0]:=36.547:
> f2[0]:=17.264:
> f3[0]:=8.155:
> f4[0]:=3.852:
> f5[0]:=1.820:
> f6[0]:=0.86:
> f7[0]:=0.406:
> f8[0]:=0.246:
> N:=8;
> F:= vector([f1[0],f2[0], f3[0], f4[0],f5[0],f6[0],f7[0],f8[0]]);
```

$$N := 8$$

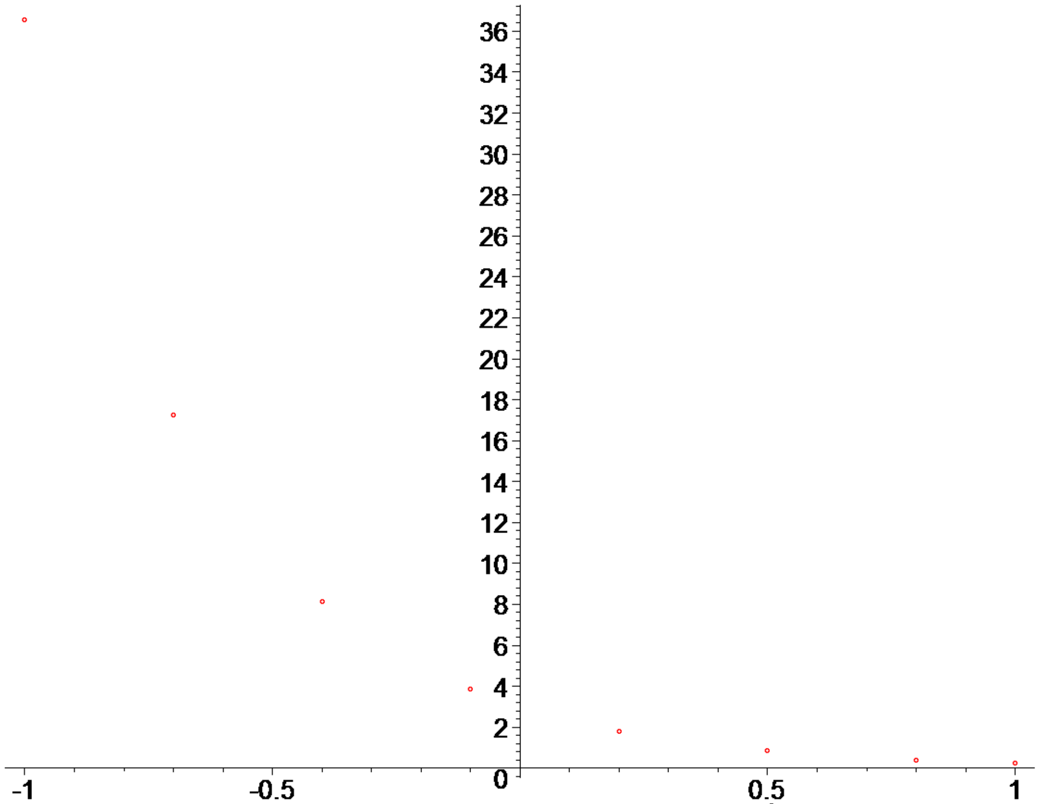
$$F := [36.547, 17.264, 8.155, 3.852, 1.820, .86, .406, .246]$$

```
> X:=vector([x1[0],x2[0],x3[0],x4[0],x5[0],x6[0],x7[0],x8[0]]);
>
```

$$X := [-1, -.7, -.4, -.1, .2, .5, .8, 1.0]$$

```
>
> VRAP1:= [[ X[i], F[i]] $i=1..8];
  plot(VRAP1,style=point,symbol=circle);
```

$VRAP1$:= [[-1, 36.547], [-.7, 17.264], [-.4, 8.155], [-.1, 3.852], [.2, 1.820], [.5, .86], [.8, .406], [1.0, .246]]



O DIAGRAMA SUGERE QUE A FUNÇÃO A SER ESCOLHIDA É UMA EXPONENCIAL DA FORMA
Phi(x) = a*exp^{-bx}. G(x)=ln(Phi(x))=ln(a)-bx=alpha_1+alpha_2*x
PROCESSO DE LINEARIZAÇÃO g(x)=ln(f(x))

```
> g1[0]:=ln(36.547):
> g2[0]:=ln(17.264):
> g3[0]:=ln(8.155):
> g4[0]:=ln(3.852):
> g5[0]:=ln(1.820):
> g6[0]:=ln(0.86):
> g7[0]:=ln(0.406):
> g8[0]:=ln(0.246):
> N:=8;
> G:= vector([g1[0],g2[0], g3[0], g4[0],g5[0],g6[0],g7[0],g8[0]]);
```

$N := 8$

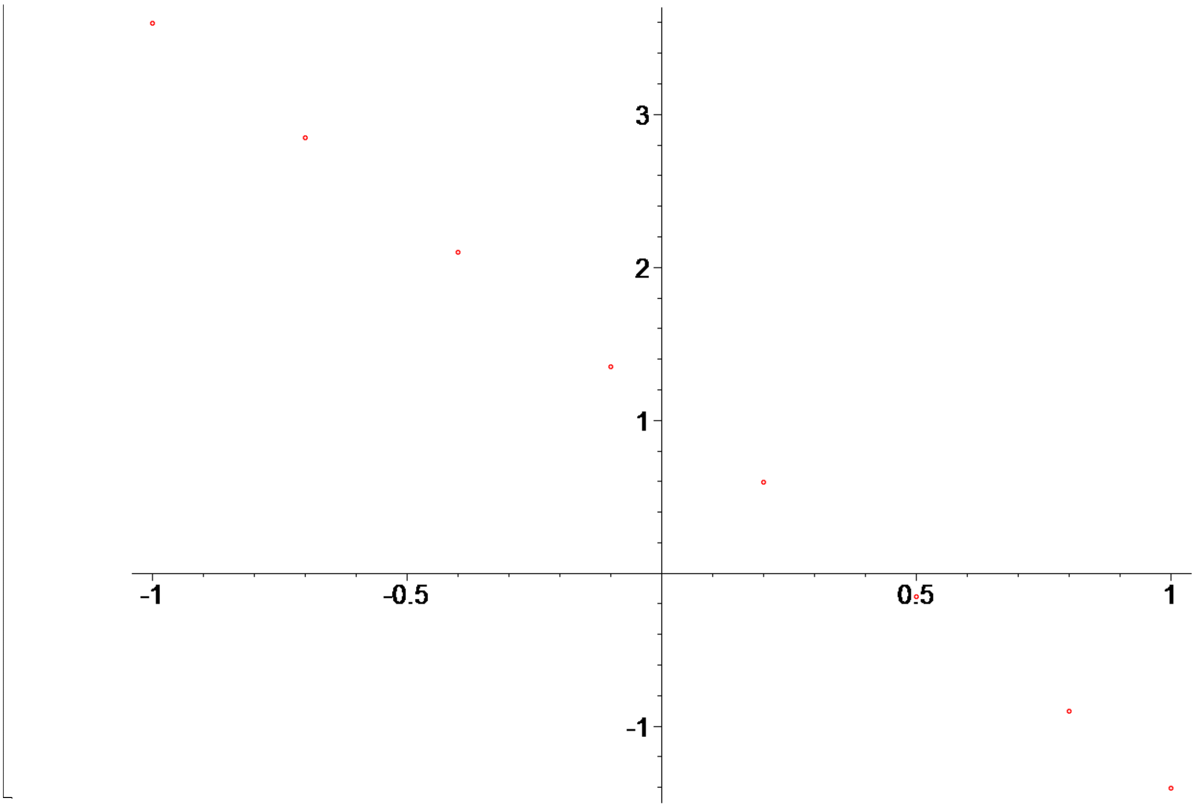
$G := [3.598599103, 2.848623409, 2.098631236, 1.348592494, .5988365011, -.1508228897, -.9014021194, -1.402423743]$

```
> X:=vector([x1[0],x2[0],x3[0],x4[0],x5[0],x6[0],x7[0],x8[0]]);
```

$X := [-1, -.7, -.4, -.1, .2, .5, .8, 1.0]$

```
> VRAP2:= [[ X[i], G[i]] $i=1..8];
  plot(VRAP2,style=point,symbol=circle);
```

$VRAP2 := [[-1, 3.598599103], [-.7, 2.848623409], [-.4, 2.098631236], [-.1, 1.348592494], [.2, .5988365011], [.5, -.1508228897], [.8, -.9014021194], [1.0, -1.402423743]]$



CALCULANDO INICIALMENTE A RETA QUE MELHOR AJUSTA OS PONTOS

```
> X[1]:=
  array(1..8,[x1[0],x2[0],x3[0],x4[0],x5[0],x6[0],x7[0],x8[0]]):;
> a11:=w1[0]+w2[0]+w3[0]+w4[0]+w5[0]+w6[0]+w7[0]+w8[0]:;
> a12:=w1[0]*x1[0]+w2[0]*x2[0]+w3[0]*x3[0]+w4[0]*x4[0]+w5[0]*x5[0]
  +w6[0]*x6[0]+w7[0]*x7[0]+w8[0]*x8[0]:;
> a21:=a12:
> a22:=w1[0]*x1[0]^2+w2[0]*x2[0]^2+w3[0]*x3[0]^2+w4[0]*x4[0]^2+w5[
  0]*x5[0]^2+w6[0]*x6[0]^2+w7[0]*x7[0]^2+w8[0]*x8[0]^2:;
> b1:=w1[0]*g1[0]+w2[0]*g2[0]+w3[0]*g3[0]+w4[0]*g4[0]+w5[0]*g5[0]+
  w6[0]*g6[0]+w7[0]*g7[0]+w8[0]*g8[0]:;
> b2:=w1[0]*g1[0]*x1[0]+w2[0]*g2[0]*x2[0]+w3[0]*g3[0]*x3[0]+w4[0]*
  g4[0]*x4[0]+w5[0]*g5[0]*x5[0]+w6[0]*g6[0]*x6[0]+w7[0]*g7[0]*x7[0
  ]+w8[0]*g8[0]*x8[0]:;
```

REGRESSÃO LINEAR: APROXIMANDO OS PONTOS PELO POLINÔMIO DE GRAU 1.

```
> A1:=matrix(2,2,[[a11,a12],[a21,a22]]);
```

$$A1 := \begin{bmatrix} 8 & .3 \\ .3 & 3.59 \end{bmatrix}$$

```
> B1:=vector(2,[b1,b2]);
> det(A1);
```

$$B1 := [8.038633988, -8.646136816]$$

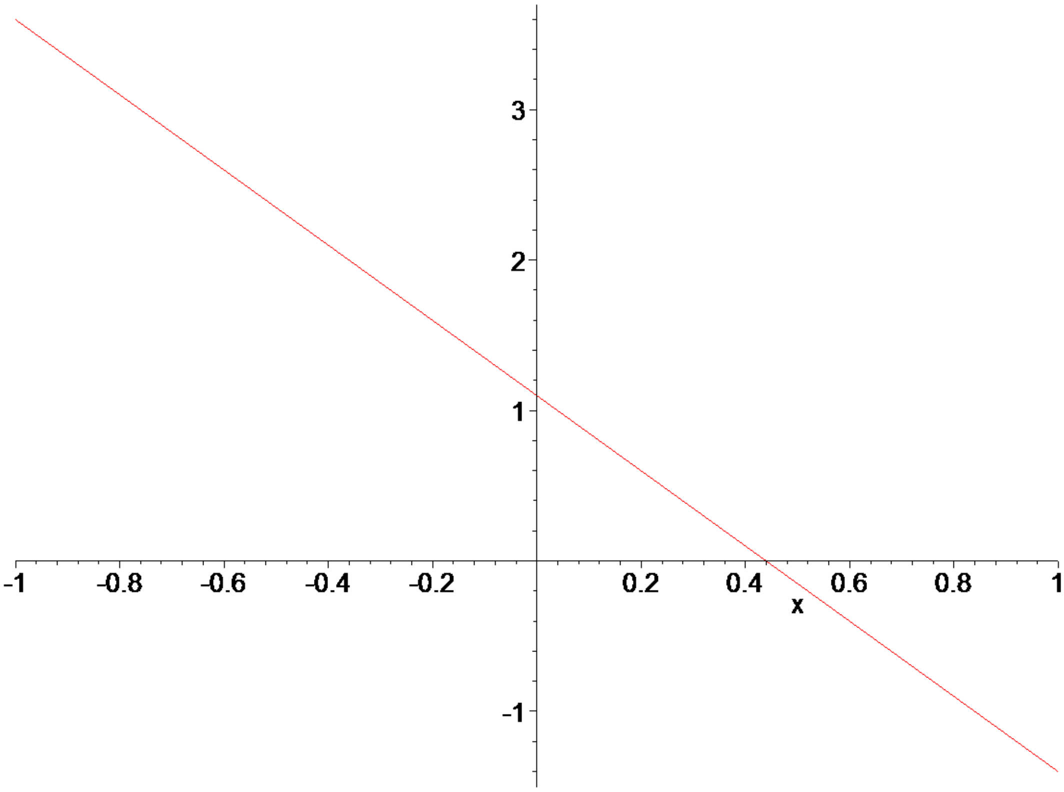
```
> C1:=linsolve(A1,B1);
```

$$C1 := [1.098586695, -2.500198559]$$

```
>
> phi1(x):=(C1[1]+C1[2]*x);
```

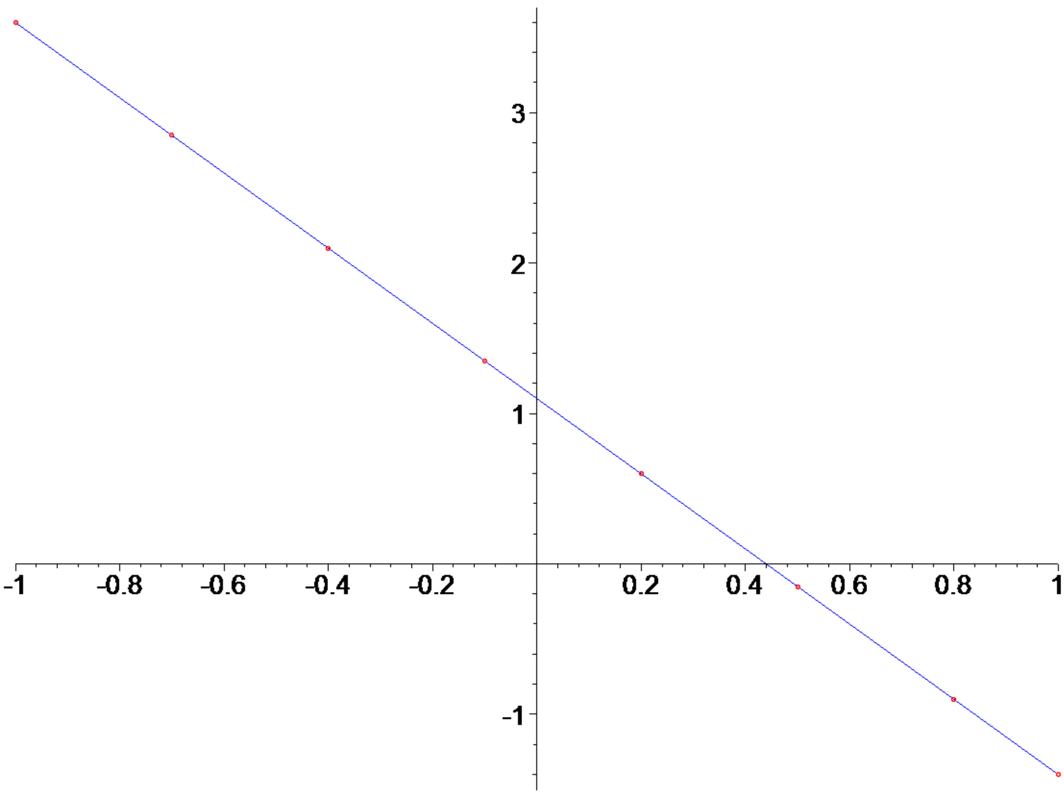
$$\phi 1(x) := 1.098586695 - 2.500198559\, x$$

```
>
> plot([phi1(x)], x=-1.0..1.0, color=[red], style=[line]);
```




```
> gr1:=plot(phi1(x), x=-1.0..1.0, color=[blue],style=[line]):

> display(plot([VRAP2],style=[point],symbol=circle),gr1);
```



CALCULO DO FUNÇÃO EXPONENCIAL

```
> a:=exp(C1[1]);
```

$$a := 2.999923220$$

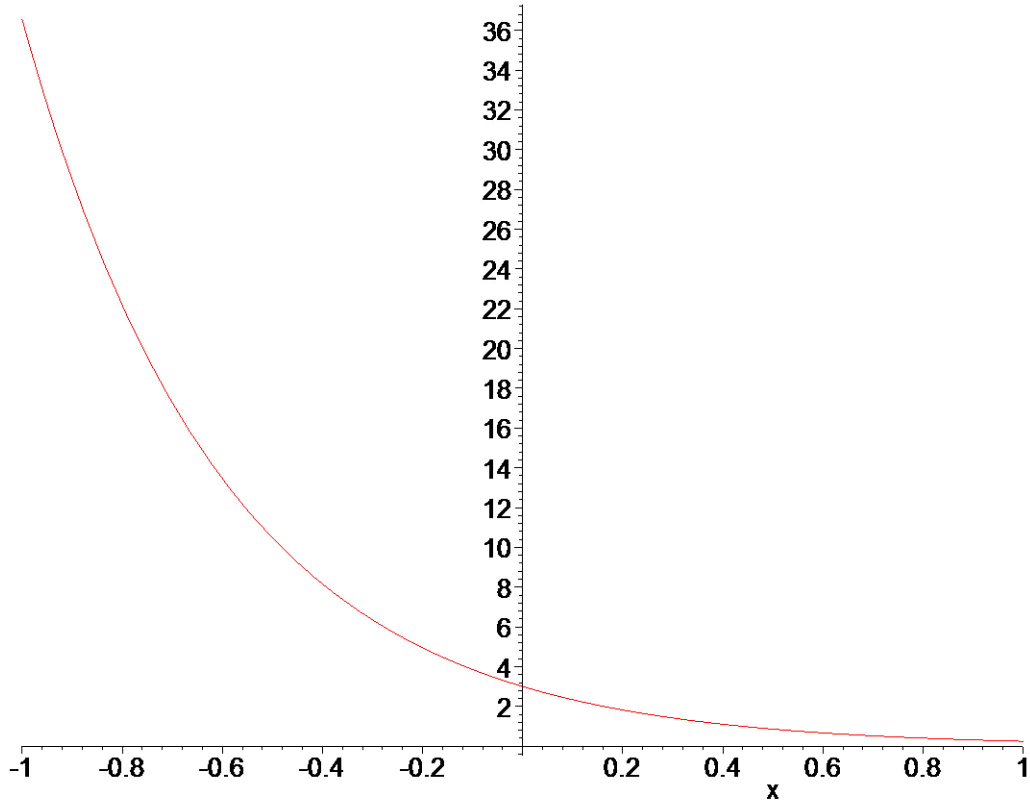
```
> b:=-C1[2];
```

$$b := 2.500198559$$

```
> w(x):=a*exp(-b*x);
```

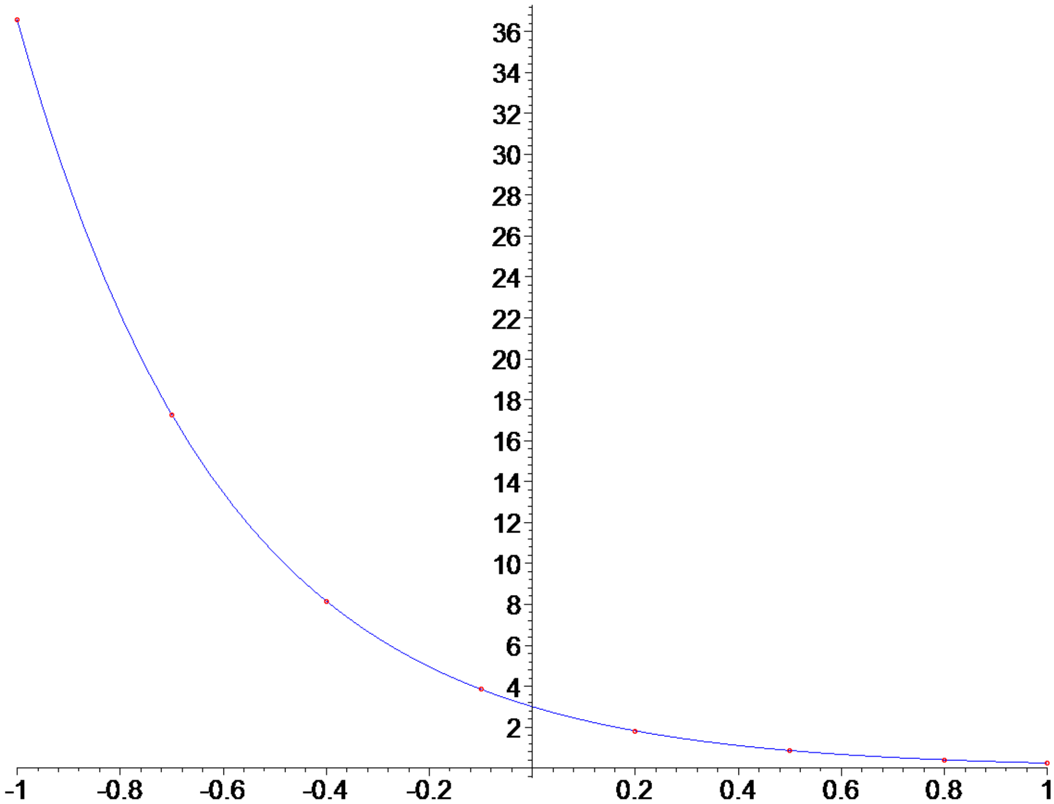
$$w(x) := 2.999923220\ \mathbf{e}^{(-2.500198559\ x)}$$

```
> plot([w(x)], x=-1.0..1.0, color=[red], style=[line]);
```




```
> gr2:=plot(w(x), x=-1.0..1.0, color=[blue],style=[line]):
```

```
> display(plot([VRAP1],style=[point],symbol=circle),gr2);
```




```
>
```

HIPÉRBOLE ; Phi(x)= 1/(a + b*x)

```
> x1[0]:=-3:
```

```
> x2[0]:=-2.0:
> x3[0]:=-1.1:
> x4[0]:=-0.9:
> x5[0]:=-0.8:
> x6[0]:=1.0:
> x7[0]:=1.5:
> x8[0]:=2.0:
```

COLOCANDO OS PESOS W_i UNIFORMES

```
> w1[0]:=1:;
> w2[0]:=1:;
> w3[0]:=1:;
> w4[0]:=1:;
> w5[0]:=1:;
> w6[0]:=1:;
> w7[0]:=1:;
> w8[0]:=1:;
```

VALOR DA FUNÇÃO NOS PONTOS

```
> f1[0]:=-0.5:
> f2[0]:=-1.0:
> f3[0]:=-5.0:
> f4[0]:=-10.0:
> f5[0]:=4.0:
> f6[0]:=1.0:
> f7[0]:=0.7:
> f8[0]:=0.5:
> N:=8;
> F:= vector([f1[0],f2[0], f3[0], f4[0],f5[0],f6[0],f7[0],f8[0]]);
```

$$N := 8$$

$$F := [-.5, -1.0, -5.0, -10.0, 4.0, 1.0, .7, .5]$$

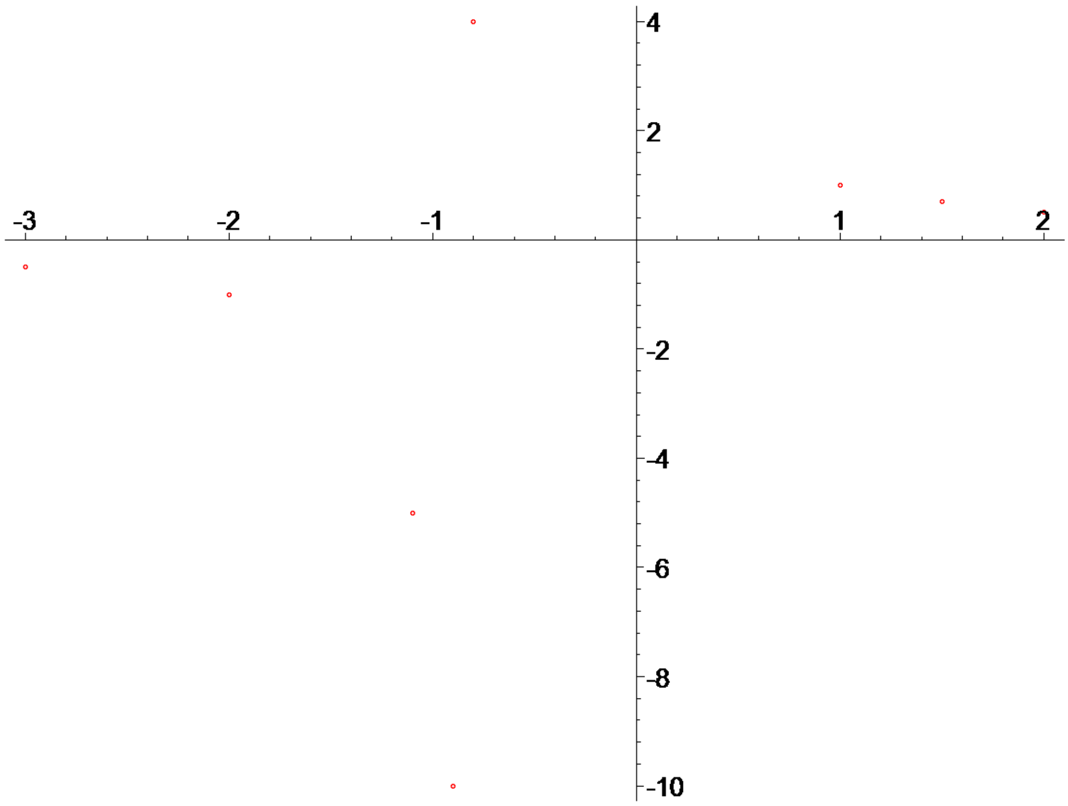
```
> X:=vector([x1[0],x2[0],x3[0],x4[0],x5[0],x6[0],x7[0],x8[0]]);
>
```

$$X := [-3, -2.0, -1.1, -.9, -.8, 1.0, 1.5, 2.0]$$

```
> VRAP3:= [[ X[i], F[i]] $i=1..8];
  plot(VRAP3,style=point,symbol=circle);
```

$VRAP3 :=$

$$[[-3, -.5], [-2.0, -1.0], [-1.1, -5.0], [-.9, -10.0], [-.8, 4.0], [1.0, 1.0], [1.5, .7], [2.0, .5]]$$



U(x) = 1/Phi(x) = (a + b*x)

```
> g1[0]:=1/f1[0]:
> g2[0]:=1/f2[0]:
> g3[0]:=1/f3[0]:
> g4[0]:=1/f4[0]:
> g5[0]:=1/f5[0]:
> g6[0]:=1/f6[0]:
> g7[0]:=1/f7[0]:
> g8[0]:=1/f8[0]:
> N:=8;
> G:= vector([g1[0],g2[0], g3[0], g4[0],g5[0],g6[0],g7[0],g8[0]]);
```

$N := 8$

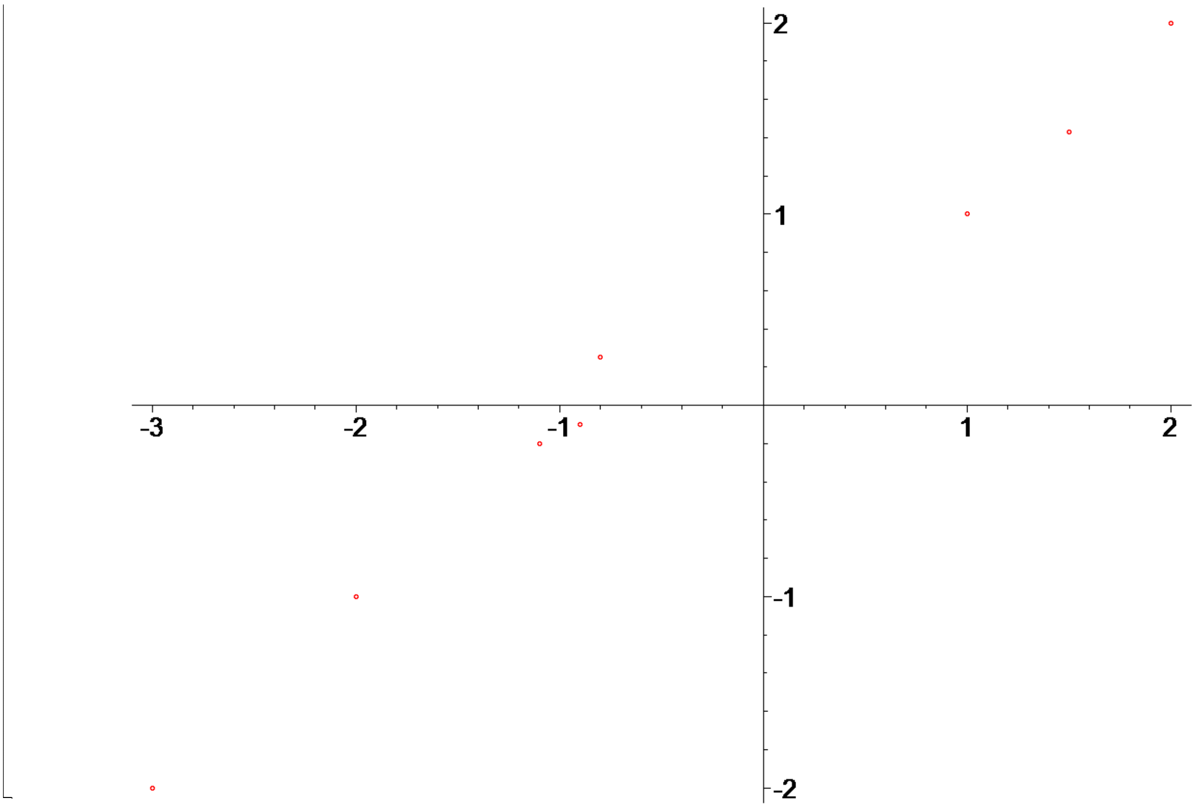
$G := [-2.000000000, -1.000000000, -.2000000000, -.1000000000, .2500000000, 1.000000000, 1.428571429, 2.000000000]$

```
> X:=vector([x1[0],x2[0],x3[0],x4[0],x5[0],x6[0],x7[0],x8[0]]);
```

$X := [-3, -2.0, -1.1, -.9, -.8, 1.0, 1.5, 2.0]$

```
>
> VRAP4:= [[ X[i], G[i]] $i=1..8];
  plot(VRAP4,style=point,symbol=circle);
```

$VRAP4 := [[-3, -2.000000000], [-2.0, -1.000000000], [-1.1, -.2000000000], [-.9, -.1000000000], [-.8, .2500000000], [1.0, 1.000000000], [1.5, 1.428571429], [2.0, 2.000000000]]$



OBSERVE QUE O DIAGRAMA DA INVERSA PODE SER APROXIMADO POR UMA RETA
Phi(x)=alpha_1 +alpha_2*x
Note que iremos aproximar(x,g(x))=(x,1/f(x))

```
> X[1]:=
  array(1..8,[x1[0],x2[0],x3[0],x4[0],x5[0],x6[0],x7[0],x8[0]]):;
> a11:=w1[0]+w2[0]+w3[0]+w4[0]+w5[0]+w6[0]+w7[0]+w8[0]:;
> a12:=w1[0]*x1[0]+w2[0]*x2[0]+w3[0]*x3[0]+w4[0]*x4[0]+w5[0]*x5[0]
  +w6[0]*x6[0]+w7[0]*x7[0]+w8[0]*x8[0]:;
> a21:=a12:
> a22:=w1[0]*x1[0]^2+w2[0]*x2[0]^2+w3[0]*x3[0]^2+w4[0]*x4[0]^2+w5[
  0]*x5[0]^2+w6[0]*x6[0]^2+w7[0]*x7[0]^2+w8[0]*x8[0]^2:;
> b1:=w1[0]*g1[0]+w2[0]*g2[0]+w3[0]*g3[0]+w4[0]*g4[0]+w5[0]*g5[0]+
  w6[0]*g6[0]+w7[0]*g7[0]+w8[0]*g8[0]:;
> b2:=w1[0]*g1[0]*x1[0]+w2[0]*g2[0]*x2[0]+w3[0]*g3[0]*x3[0]+w4[0]*
  g4[0]*x4[0]+w5[0]*g5[0]*x5[0]+w6[0]*g6[0]*x6[0]+w7[0]*g7[0]*x7[0
  ]+w8[0]*g8[0]*x8[0]:;
```

REGRESSÃO LINEAR: APROXIMANDO OS PONTOS PELO POLINÔMIO DE GRAU 1.

```
> A2:=matrix(2,2,[[a11,a12],[a21,a22]]);
```

$$A2 := \begin{bmatrix} 8 & -3.3 \\ -3.3 & 22.91 \end{bmatrix}$$

```
> B2:=vector(2,[b1,b2]);
> det(A2);
```

$$B2 := [\, 1.378571429, 15.25285714 \,]$$

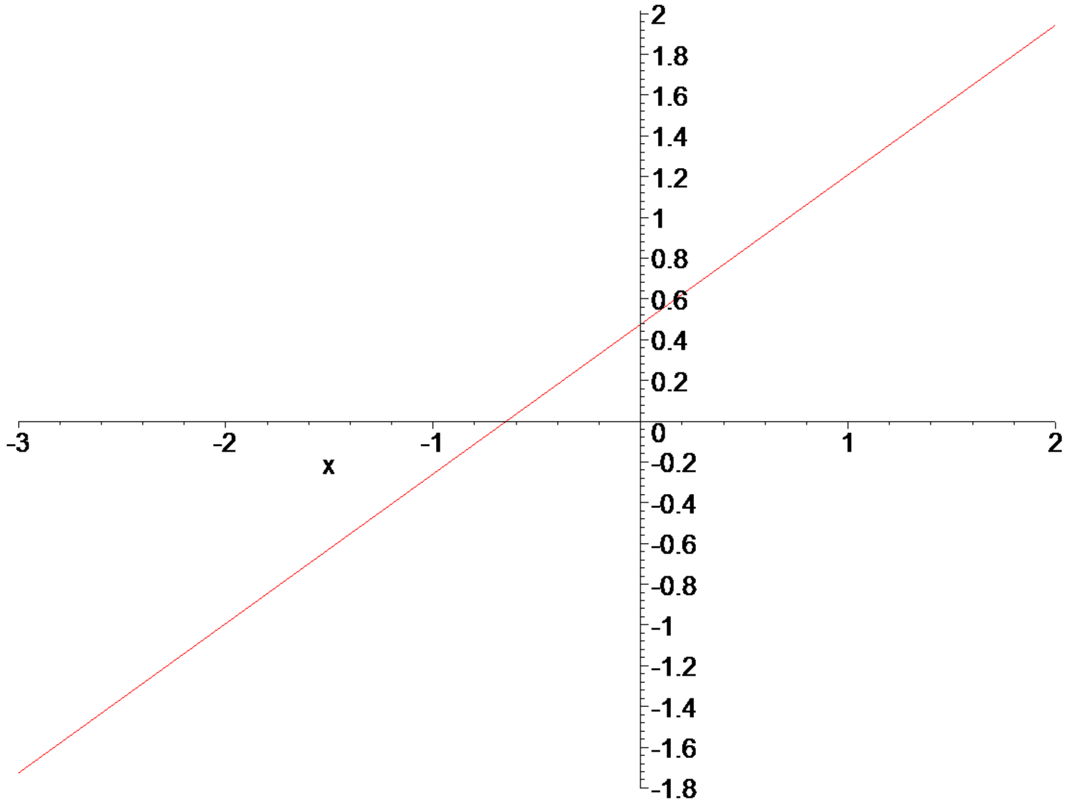
```
> C2:=linsolve(A2,B2);
```

$$C2 := [\,.4751870758,\ .7342197508\,]$$

```
> phi2(x):=(C2[1]+C2[2]*x);
```

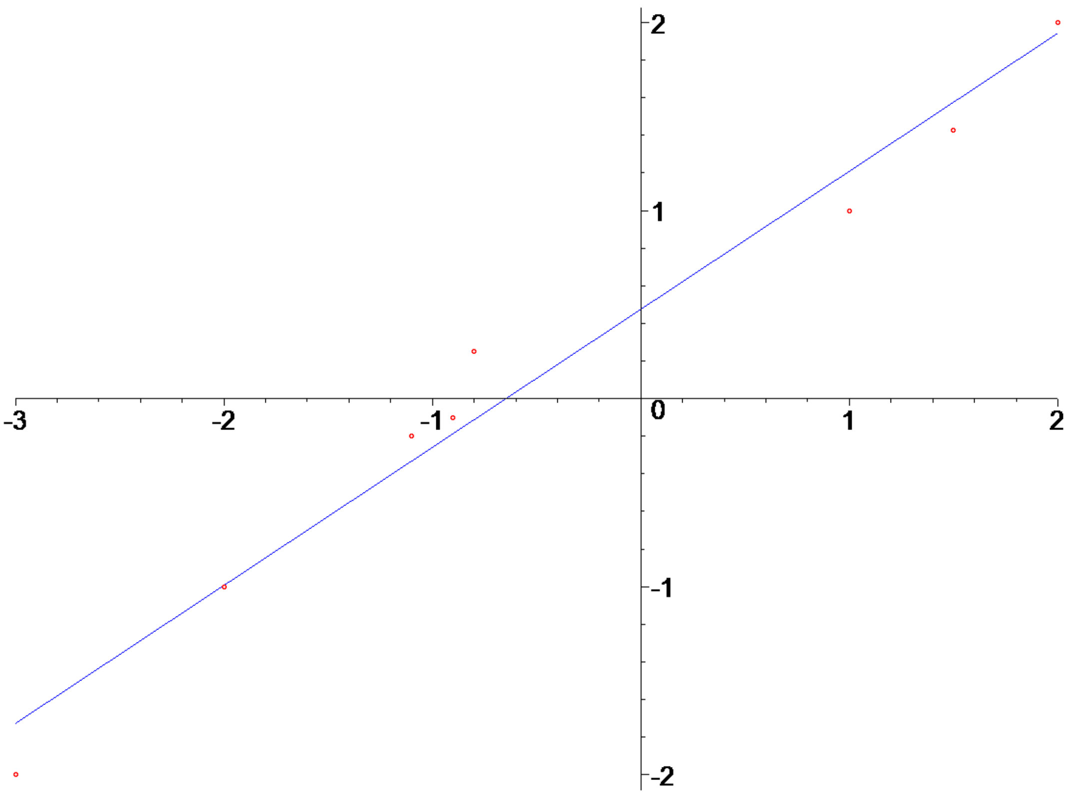
$$\phi 2(x) := .4751870758 + .7342197508\,x$$

```
> plot([phi2(x)], x=-3.0..2.0, color=[red], style=[line]);
```




```
> gr2:=plot(phi2(x), x=-3.0..2.0, color=[blue],style=[line]):
```

```
> display(plot([VRAP4],style=[point],symbol=circle),gr2);
```



CALCULO DA HIPÉRBOLE

```
> a1:=C2[1];
```

$$a1 := .4751870758$$

```
> a2:=C2[2];
```

$$a2 := .7342197508$$

```
> phi2(x):=(C2[1]+C2[2]*x);
```

$$\phi 2(x) := .4751870758 + .7342197508\, x$$

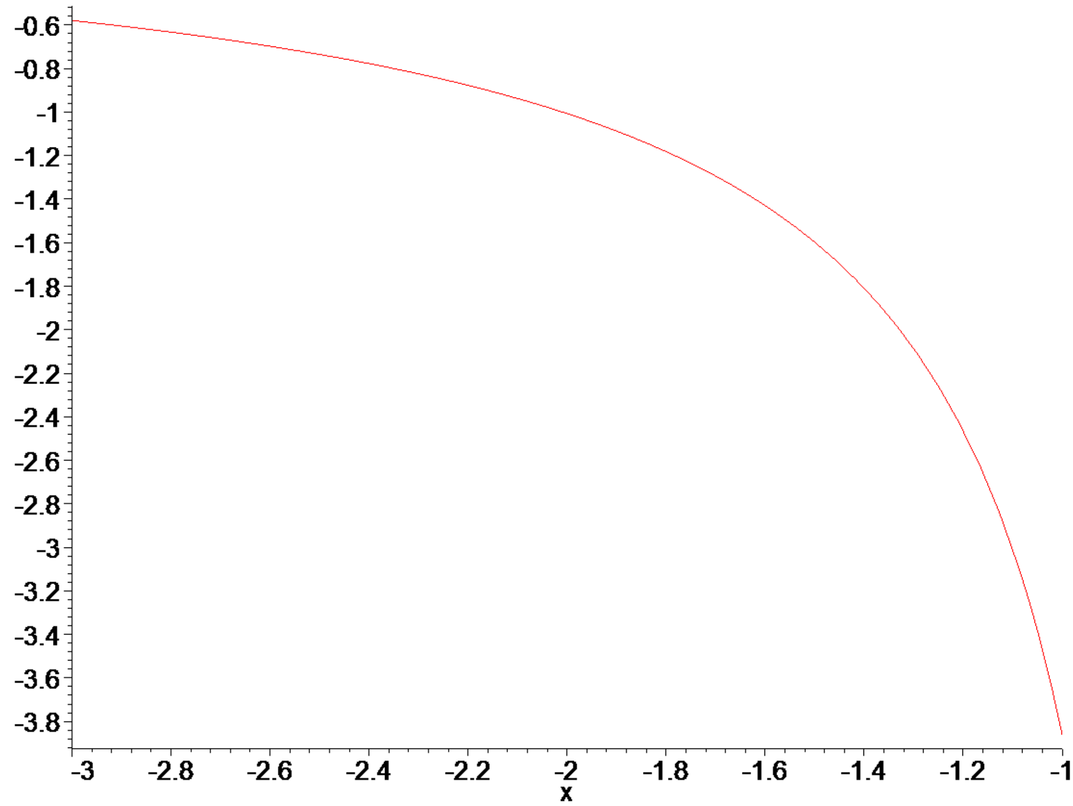
```
> phi3(x):=1/phi2(x);
```

$$\phi 3(x) := \frac{1}{.4751870758 + .7342197508\, x}$$

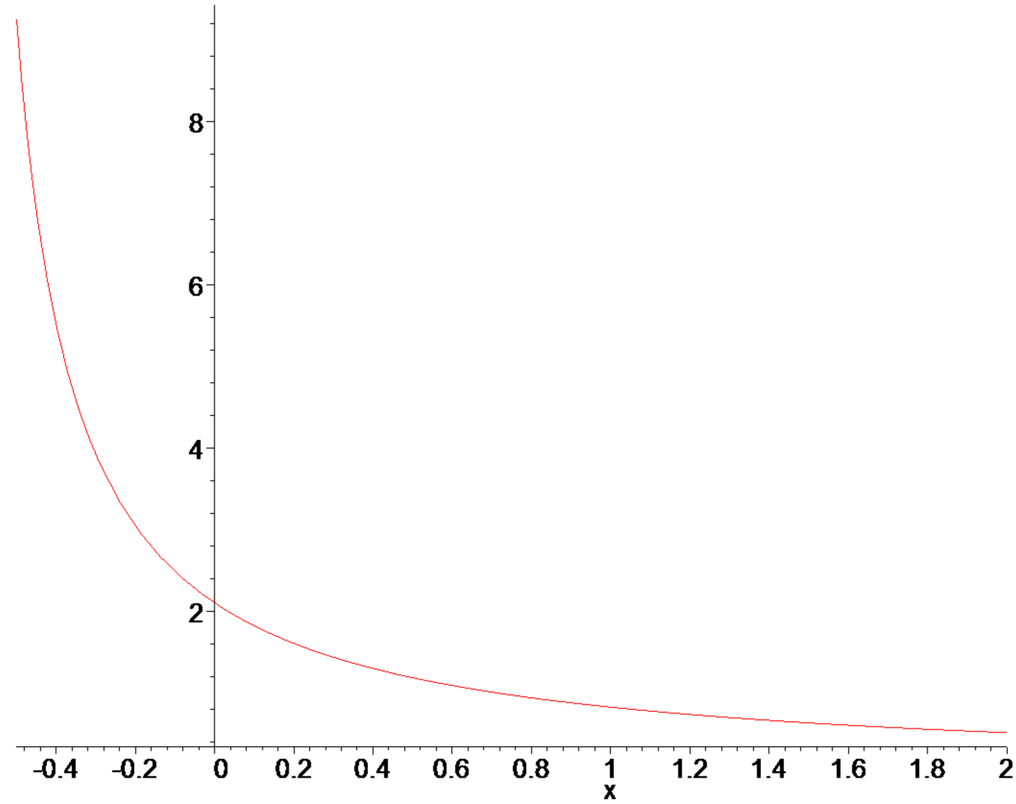
```
> z:=-C2[1]/C2[2];
```

$$z := -.6472000723$$

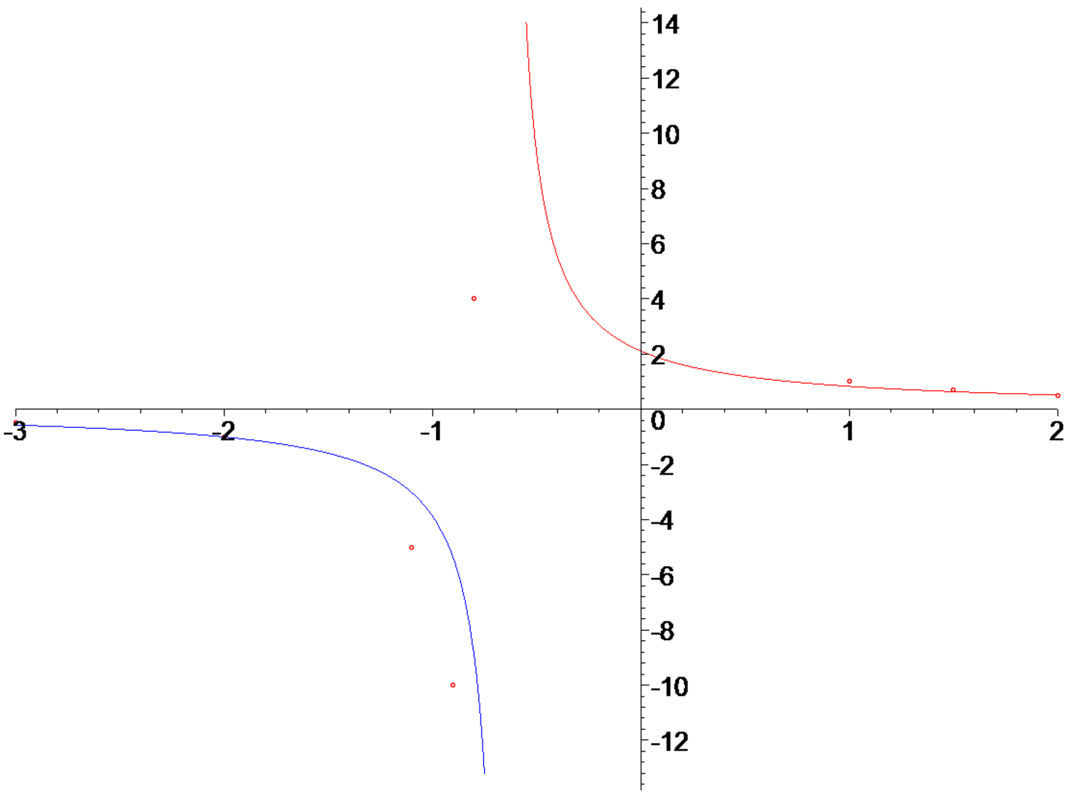
```
> plot([phi3(x)], x=-3.0..-1.0, color=[red], style=[line]);
```




```
> plot([phi3(x)], x=-0.5..2.0, color=[red], style=[line]);
```




```
> gr6:=plot(phi3(x), x=-3.0..-0.75, color=[blue],style=[line]):
> gr7:=plot(phi3(x), x=-0.55..2.0, color=[red],style=[line]):
> display(plot([VRAP3],style=[point],symbol=circle),gr6,gr7);
```



CURVA GEOMÉTRICA w(x)=a*x^b, b=2,3,4
ln(w(x)=ln(a)+b*(ln(x), observe que nesse caso varphi_1(x)=1., mas varphi_2(x)=ln(x)
v(y)=alpha + beta*y

```
>
> x1[0]:=0.5:
> x2[0]:=0.7:
> x3[0]:=0.9:
> x4[0]:=1.1:
> x5[0]:=1.5:
> x6[0]:=1.7:
> x7[0]:=2.1:
> x8[0]:=2.9:
> f1[0]:=0.24:;
> f2[0]:=0.75:
> f3[0]:=3.25:
> f4[0]:=3.5:
> f5[0]:=9.9:
> f6[0]:=15.20:
> f7[0]:=31.15:
> f8[0]:=93.4:
>
> a11:=w1[0]+w2[0]+w3[0]+w4[0]+w5[0]+w6[0]+w7[0]+w8[0]:;
> a12:=w1[0]*ln(x1[0])+w2[0]*ln(x2[0])+w3[0]*ln(x3[0])+w4[0]*ln(x4
  [0])+w5[0]*ln(x5[0])+w6[0]*ln(x6[0])+w7[0]*ln(x7[0])+w8[0]*ln(x8
  [0]):;
> a21:=a12:
```

```
> a22:=w1[0]*ln(x1[0])^2+w2[0]*ln(x2[0])^2+w3[0]*ln(x3[0])^2+w4[0]
*ln(x4[0])^2+w5[0]*ln(x5[0])^2+w6[0]*ln(x6[0])^2+w7[0]*ln(x7[0])
^2+w8[0]*ln(x8[0])^2:;
> b1:=w1[0]*ln(f1[0])+w2[0]*ln(f2[0])+w3[0]*ln(f3[0])+w4[0]*ln(f4[
0])+w5[0]*ln(f5[0])+w6[0]*ln(f6[0])+w7[0]*ln(f7[0])+w8[0]*ln(f8[
0]):;
> b2:=w1[0]*ln(f1[0])*ln(x1[0])+w2[0]*ln(f2[0])*ln(x2[0])+w3[0]*ln
(f3[0])*ln(x3[0])+w4[0]*ln(f4[0])*ln(x4[0])+w5[0]*ln(f5[0])*ln(x
5[0])+w6[0]*ln(f6[0])*ln(x6[0])+w7[0]*ln(f7[0])*ln(x7[0])+w8[0]*
ln(f8[0])*ln(x8[0]):;
>
```

REGRESSÃO LINEAR: APROXIMANDO OS PONTOS PELO POLINÔMIO DE GRAU 1.

```
> A3:=matrix(2,2,[[a11,a12],[a21,a22]]);
```

$$A3 := \begin{bmatrix} 8 & 1.682868980 \\ 1.682868980 & 2.757903170 \end{bmatrix}$$

```
> B3:=vector(2,[b1,b2]);
> det(A3);
```

$$B3 := [\,13.70615531,\ 10.84242876\,]$$

$$19.23117736$$

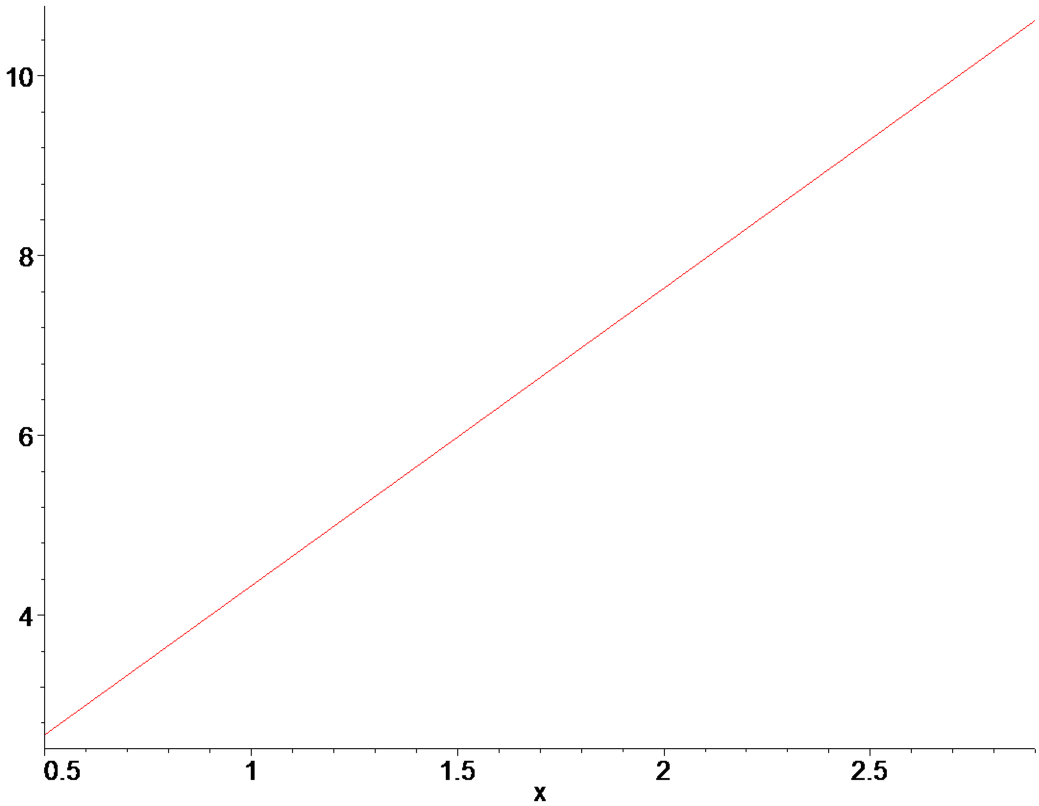
```
> C3:=linsolve(A3,B3);
```

$$C3 := [\,1.016779254,\ 3.310965589\,]$$

```
>
> phi3(x):=(C3[1]+C3[2]*x);
```

$$\phi 3(x) := 1.016779254 + 3.310965589\ x$$

```
> plot([phi3(x)], x=0.5..2.9, color=[red], style=[line]);
```



DA FORMA GEOMETRICA: y=a*x^b. Aplicando o logaritmo neperiano tem se ln(y)=ln(a)+b*ln(x) que pode ser escrito
z=alpha+b*t. Calculada a reta Z então encontra-se a forma geométrica fazendo a mudança;
a=exp(alpha) , b=b

```
> alpha:=exp(C3[1]);
```

$$\alpha := 2.764277376$$

```
> H(x):=alpha*x^(C3[2]);
```

$$\mathrm{H}(x) := 2.764277376\, x^{3.310965589}$$

```
> gr8:=plot([H(x)], x=0.5..2.9, color=[red], style=[line]):
> N:=8;
> F:= vector([f1[0],f2[0], f3[0], f4[0],f5[0],f6[0],f7[0],f8[0]]);
```

$$N := 8$$

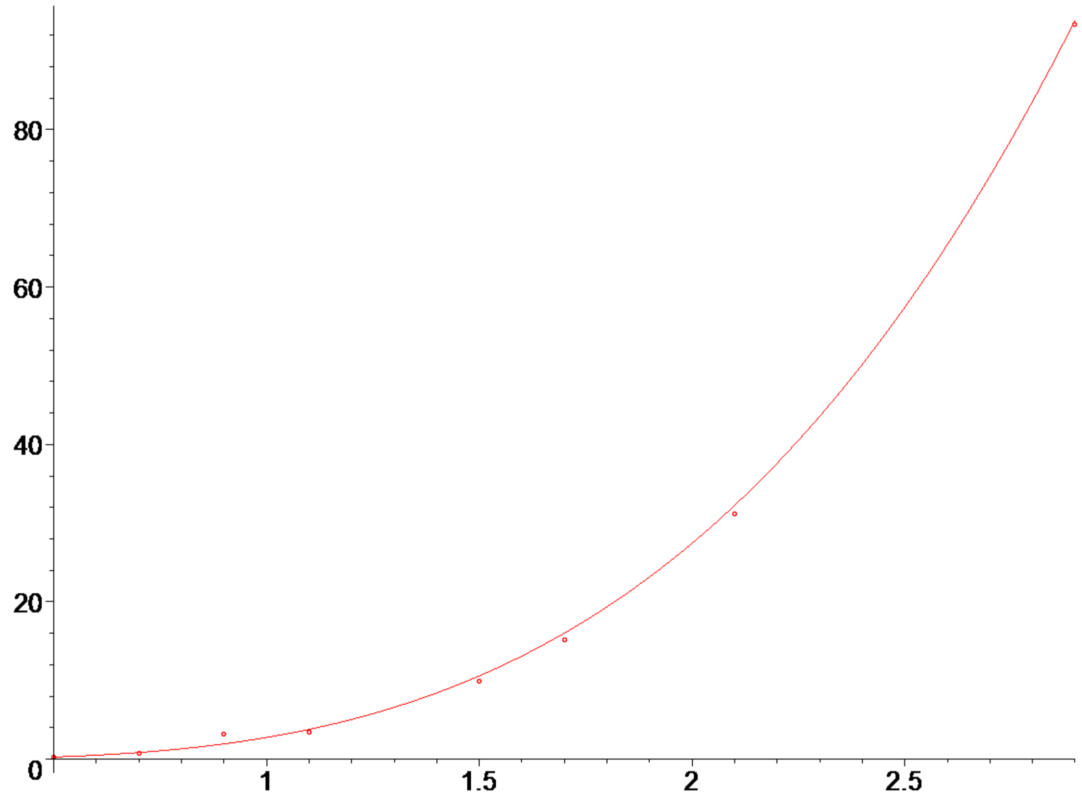
$$F := [.24, .75, 3.25, 3.5, 9.9, 15.20, 31.15, 93.4]$$

```
> X:=vector([x1[0],x2[0],x3[0],x4[0],x5[0],x6[0],x7[0],x8[0]]):;
> VRAP8:= [[ X[j], F[j]] $j=1..8];
  plot(VRAP8,style=point,symbol=circle):;
```

VRAP8 :=

[[.5, .24], [.7, .75], [.9, 3.25], [1.1, 3.5], [1.5, 9.9], [1.7, 15.20], [2.1, 31.15], [2.9, 93.4]]

```
> display(plot([VRAP8],style=[point],symbol=circle),gr8);
```



POLINÔMIOS ORTOGONAIS

Ajuste de curvas por polinomios de grau 1 e 2.

Recorrência: P_{i+1}=(x-alpha{i+1}P_{i}+beta_i P_{i-1}

POLINÔMIOS DE GRAU 1

```
> x[1]:=0.5:;
> x[2]:=0.7:
> x[3]:=0.9:
> x[4]:=1.1:
> x[5]:=1.5:
> x[6]:=1.7:
> x[7]:=2.1:
> x[8]:=2.9:
> f[1]:=0.24:;
> f[2]:=0.75:
> f[3]:=1.25:
> f[4]:=1.5:
> f[5]:=2.9:
> f[6]:=3.20:
> f[7]:=4.15:
> f[8]:=5.4:
> m:=8:
> P_0:=1;
```

```
>
> P_1:=x-(1/m)*(x[1]+x[2]+x[3]+x[4]+x[5]+x[6]+x[7]+x[8]);
```

$$P\_1 := x - 1.425000000$$

```
> b1:=f[1]+f[2]+f[3]+f[4]+f[5]+f[6]+f[7]+f[8];
```

$$b1 := 19.39$$

```
> b2:=f[1]*(x[1]-1.425)+f[2]*(x[2]-1.425)+f[3]*(x[3]-1.425)+f[4]*(
x[4]-1.425)+f[5]*(x[5]-1.425)+f[6]*(x[6]-1.425)+f[7]*(x[7]-1.425
)+f[8]*(x[8]-1.425);
```

$$b2 := 9.95425$$

```
> a22:=0:
> for i from 1 to 8 do
>
> a11:=i;
> a22:=a22+(x[i]-1.425)^2;
> od:;
> a_0:=b1/a11;
```

$$a\_0 := 2.423750000$$

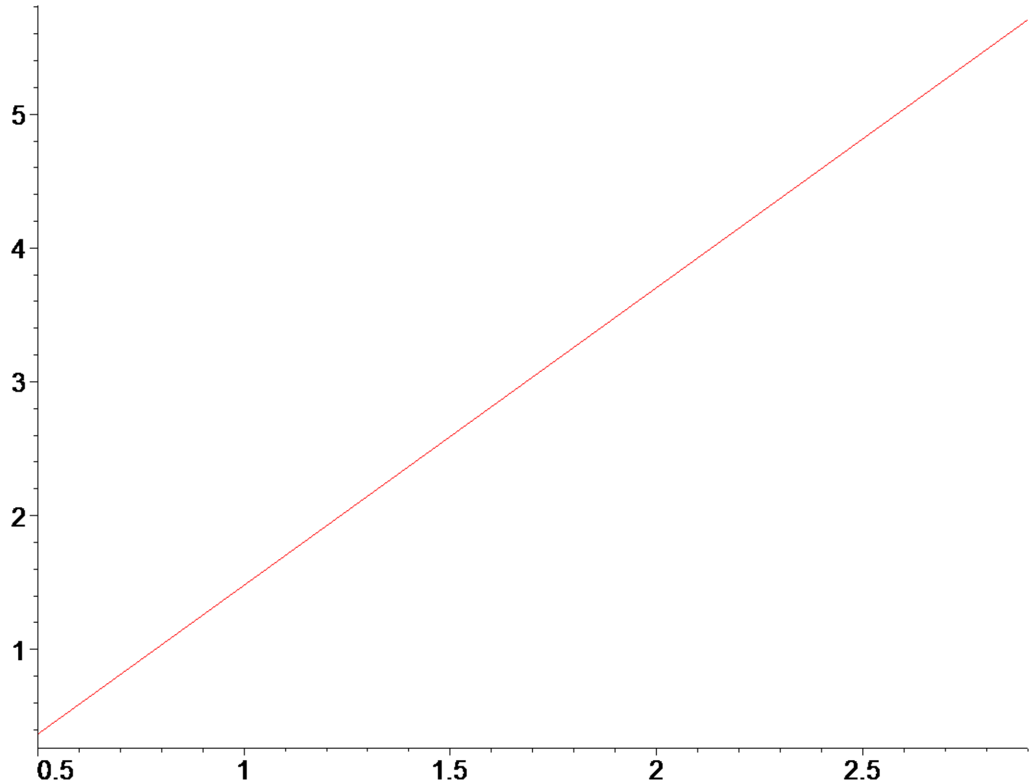
```
> a_1:=b2/a22;
```

$$a\_1 := 2.224413408$$

```
>
> Q1(x):=a_0+a_1*P_1;
```

$$\mathrm{Q1}(x) := -.746039106 + 2.224413408\, x$$

```
>
>
> z4:=plot([Q1(x)], x=0.5..2.9, color=[blue], style=[line]):
> plot([Q1(x)], x=0.5..2.9, color=[red], style=[line]);
```



```
> N:=8;
```

$$N := 8$$

```
>
> L:= vector([f[1],f[2], f[3], f[4],f[5],f[6],f[7],f[8]]);
```

$$L := [.24, .75, 1.25, 1.5, 2.9, 3.20, 4.15, 5.4]$$

```
> X:=vector([x[1],x[2],x[3],x[4],x[5],x[6],x[7],x[8]]);
```

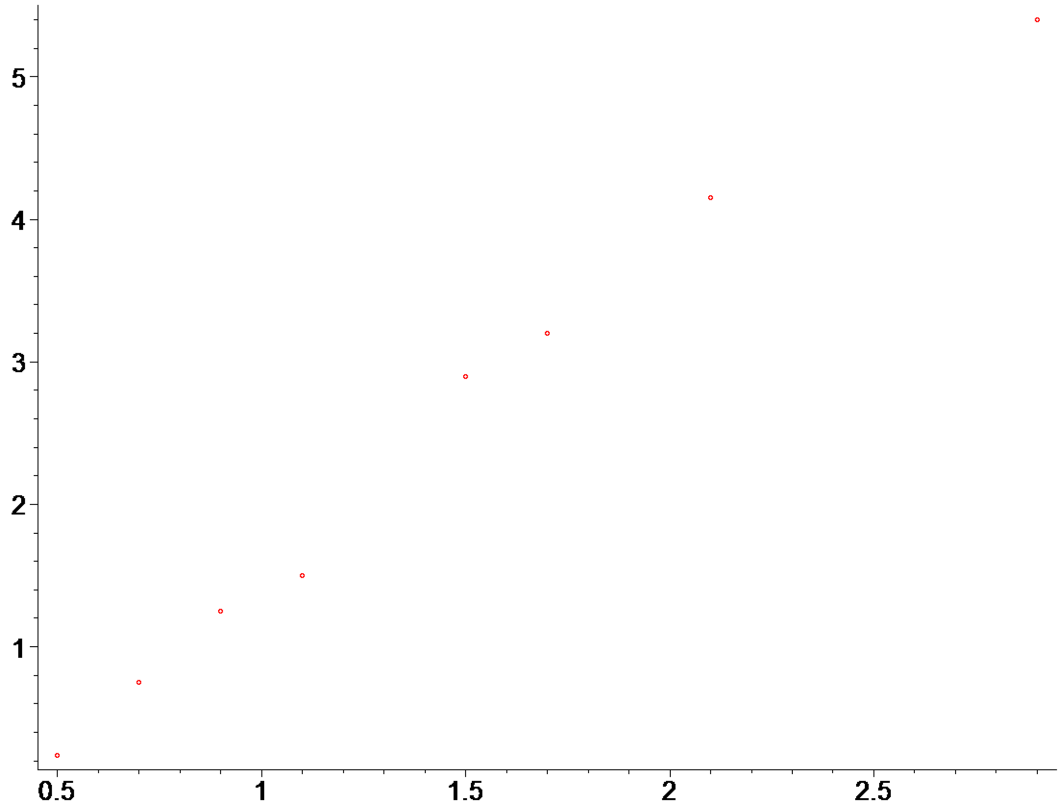
$$X := [.5, .7, .9, 1.1, 1.5, 1.7, 2.1, 2.9]$$

```
> ZZ:= [[ X[j], L[j]] $j=1..8];
```

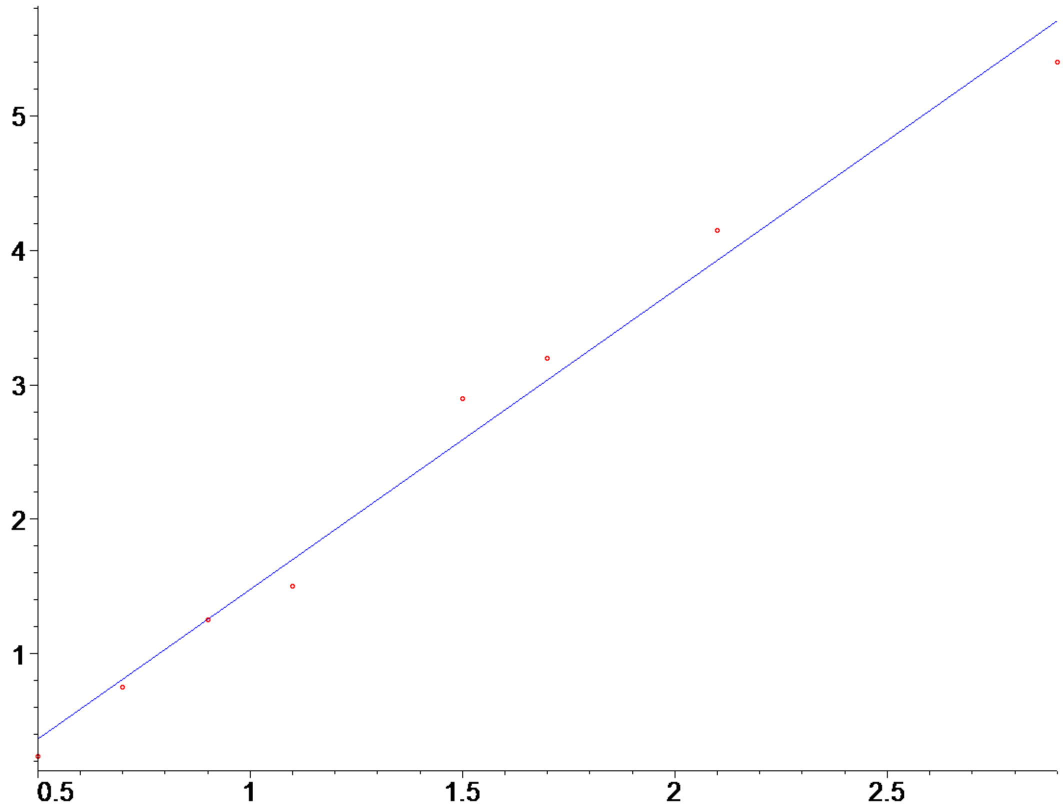
$ZZ :=$

$$[[.5, .24], [.7, .75], [.9, 1.25], [1.1, 1.5], [1.5, 2.9], [1.7, 3.20], [2.1, 4.15], [2.9, 5.4]]$$

```
> plot(ZZ,style=point,symbol=circle);
```




```
> display(plot([ZZ],style=[point],symbol=circle),z4);
```



POLINÔMIO DE GRAU 2

USANDO POLINÔMIOS ORTOGONAIS

Recorrência: P_{i+1}=(x-alpha{i+1}P_{i}+beta_i P_{i-1}

```
> BETA0:=0;
```

$$BETA0 := 0$$

```
> BETA:=BETA0;
```

$$BETA := 0$$

```
> ALPHA0:=0;
```

$$ALPHA0 := 0$$

```
> ALPHA1:=ALPHA0;
```

$$ALPHA1 := 0$$

```
> Q1 := x ->-.746039106+2.224413408*x;
```

$$Q1 := x \rightarrow -.746039106 + 2.224413408\,x$$

```
>
>
> for i from 1 to N do
> BETA0:=BETA0+(x[i]*(x[i]-1.425000000)):;
> ALPHA0:=ALPHA0+(x[i]*(x[i]-1.4250000000)^2):
> ALPHA1:=ALPHA1+(x[i]-1.425)^2:
>
>
>
> od:;
> BETA1:=(1/N)*BETA0;
```

$BETA1 := .5593750000$

```
> ALPHA2:=ALPHA0/ALPHA1;
```

$ALPHA2 := 1.913547486$

```
> P_0:=1;
```

$P\_0 := 1$

```
>
> P_1:=x-(1/m)*(x[1]+x[2]+x[3]+x[4]+x[5]+x[6]+x[7]+x[8]);
```

$P\_1 := x - 1.425000000$

```
> P_2:=(x-ALPHA2)*P_1-BETA1*P_0;
```

$P\_2 := (x - 1.913547486)(x - 1.425000000) - .5593750000$

```
> evalf(%);
```

$(x - 1.913547486)(x - 1.425000000) - .5593750000$

```
> P2 := x ->(x-1.913547486)*(x-1.425000000)-.5593750000;
```

$P2 := x \rightarrow (x - 1.913547486)(x - 1.425000000) - .5593750000$

```
>
> a33:=0:
> b3:=0:
> for i from 1 to 5 do
>
>
> a33:=a33+(P2(x[i]))^2;
> b3:=b3+f[i]*(P2(x[i]));
> od:;
>
>
> f33:=a33;
```

$f33 := 1.098737643$

```
> a_2:=b3/a33;
```

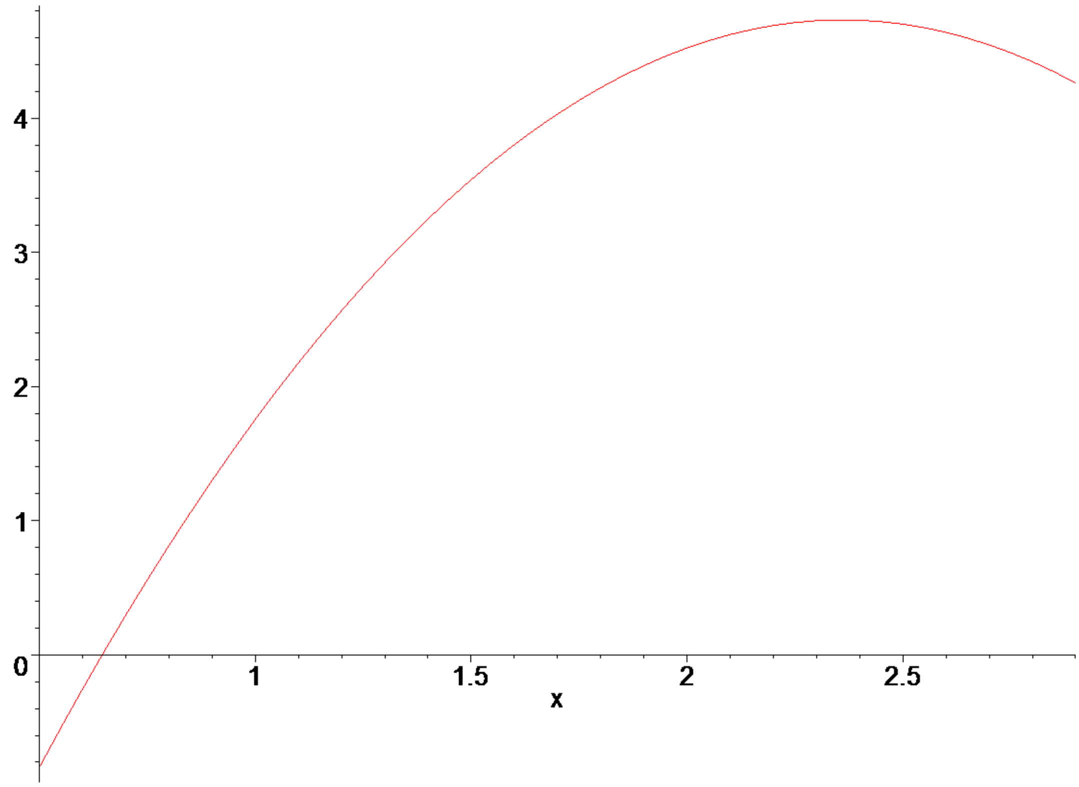
$a\_2 := -1.609827118$

```
>
> Q2(x):=a_0+a_1*P_1+a_2*P_2;
```

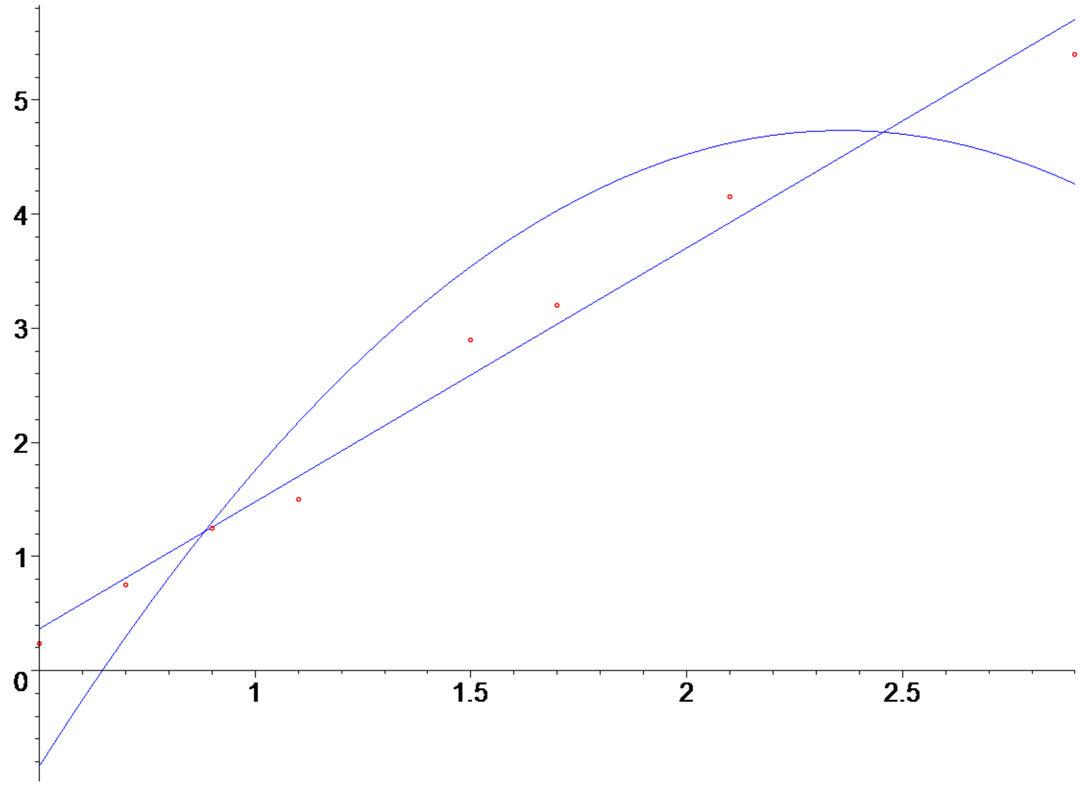
$Q2(x) :=$

$.1544579381 + 2.224413408\, x - 1.609827118\, (x - 1.913547486)(x - 1.425000000)$

```
>
>
> zb:=plot([Q2(x)], x=0.5..2.9, color=[blue], style=[line]):
>
> plot([Q2(x)], x=0.5..2.9, color=[red], style=[line]);
```




```
> display(plot([ZZ],style=[point],symbol=circle),z4,zb);
```



FAZENDO UM TESTE EM POLINOMIOS ORTOGONAIS

```
> x[1]:=0.0:;
> x[2]:=0.25:
> x[3]:=0.5:
> x[4]:=0.75:
```

```
> x[5]:=1.0:
>
> f[1]:=1.0:;
> f[2]:=2.0:
> f[3]:=1.0:
> f[4]:=0.0:
> f[5]:=1.0:
>
> m:=5:
> P_0:=1;
```

$$P\_0 := 1$$

```
>
> P_1:=x-(1/m)*(x[1]+x[2]+x[3]+x[4]+x[5]);
```

$$P\_1 := x - .5000000000$$

```
> b1:=f[1]+f[2]+f[3]+f[4]+f[5];
```

$$b1 := 5.0$$

```
> b2:=f[1]*(x[1]-0.5)+f[2]*(x[2]-0.5)+f[3]*(x[3]-0.5)+f[4]*(x[4]-0
.5)+f[5]*(x[5]-0.5);
```

$$b2 := -.500$$

```
> a22:=0:
> for i from 1 to 5 do
>
> a11:=i;
> a22:=a22+(x[i]-0.5)^2;
> od:;
> a_0:=b1/a11;
```

$$a\_0 := 1.000000000$$

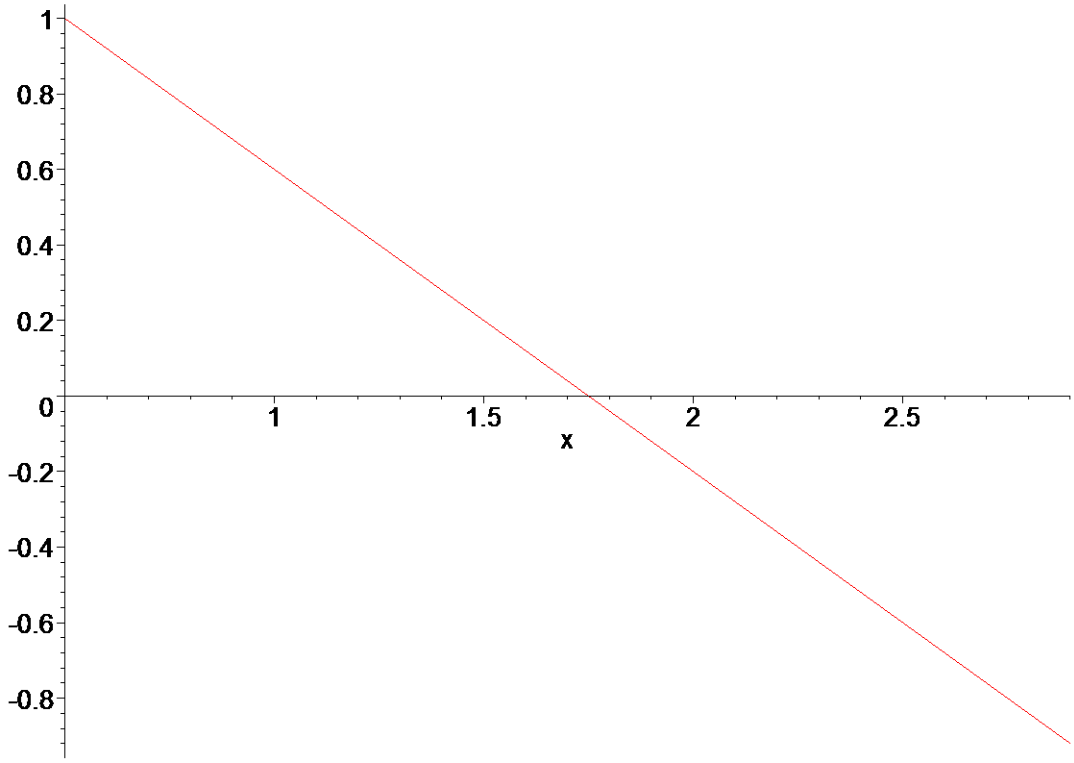
```
> a_1:=b2/a22;
```

$$a\_1 := -.8000000000$$

```
>
> Q1(x):=a_0+a_1*P_1;
```

$$\mathrm{Q1}(x) := 1.400000000 - .8000000000\, x$$

```
>
> z4:=plot([Q1(x)], x=0..1.0, color=[blue], style=[line]):
> plot([Q1(x)], x=0.5..2.9, color=[red], style=[line]);
```




```
> N:=5;
```

$$N := 5$$

```
>
> L:= vector([f[1],f[2], f[3], f[4],f[5]]);
```

$$L := [1.0, 2.0, 1.0, 0, 1.0]$$

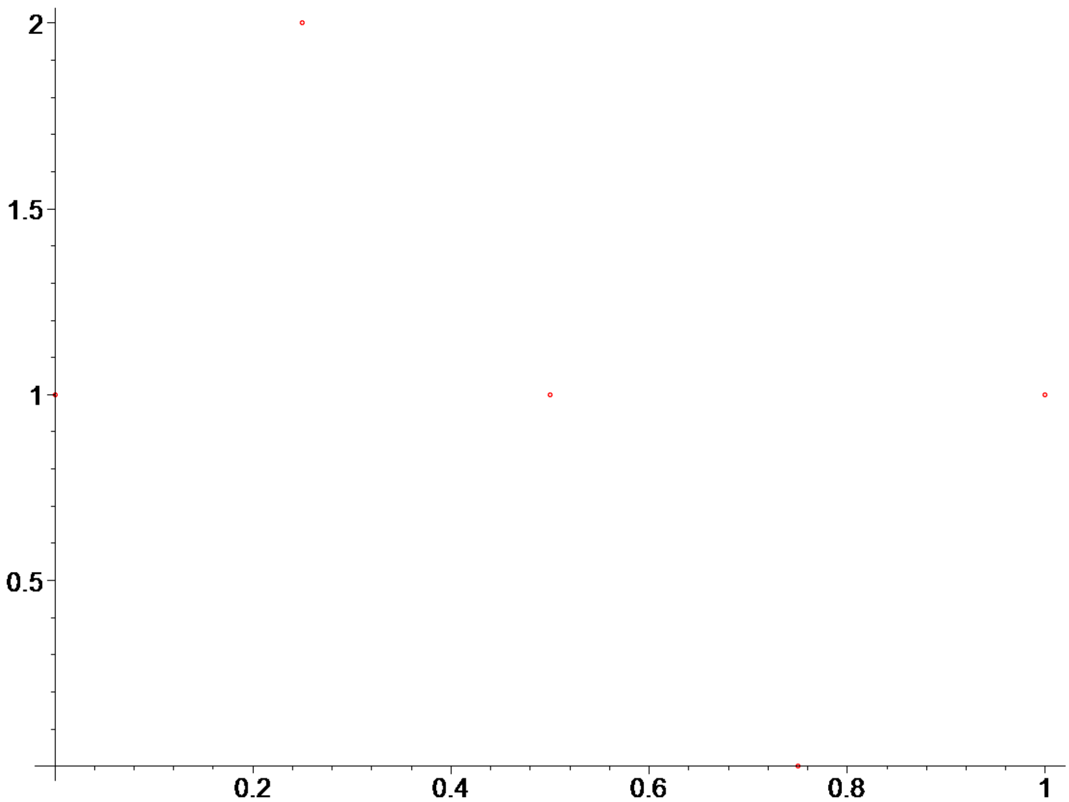
```
> X:=vector([x[1],x[2],x[3],x[4],x[5]]);
```

$$X := [0, .25, .5, .75, 1.0]$$

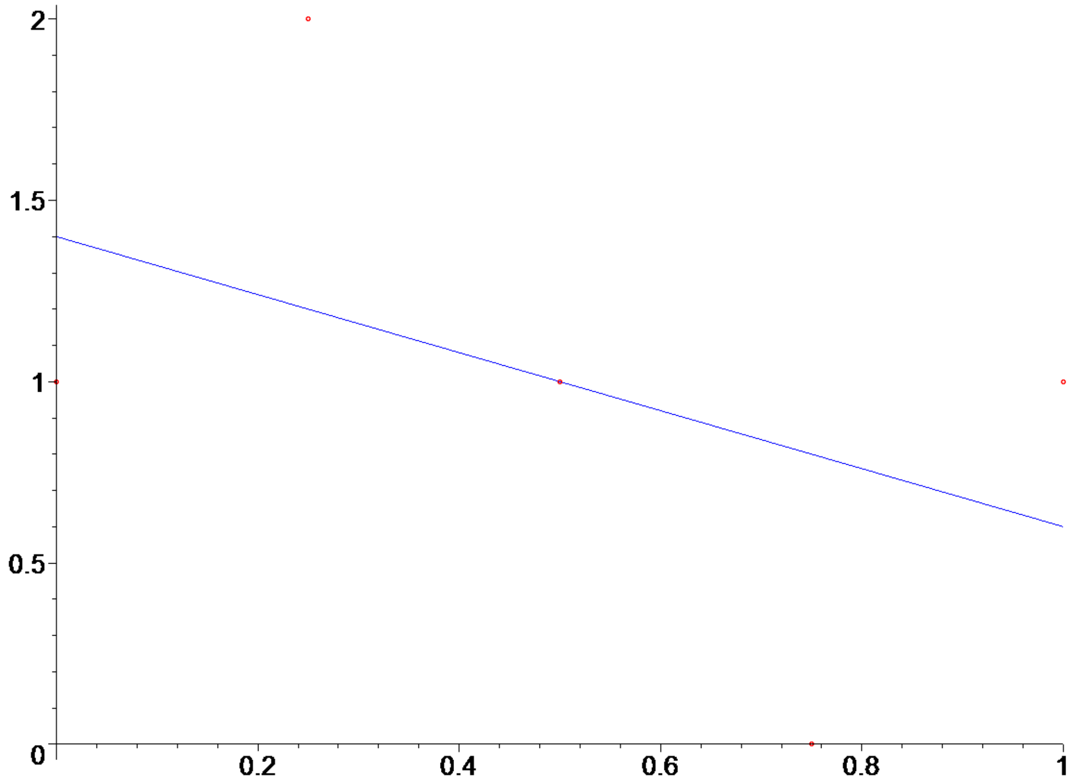
```
> ZZ1:= [[ X[j], L[j]] $j=1..5];
```

$$ZZ1 := [[0, 1.0], [.25, 2.0], [.5, 1.0], [.75, 0], [1.0, 1.0]]$$

```
> plot(ZZ1,style=point,symbol=circle);
```




```
> display(plot([ZZ1],style=[point],symbol=circle),z4);
```



POLINÔMIO DE GRAU 2

USANDO POLINÔMIOS ORTOGONAIS

Recorrência: P_{i+1}=(x-alpha{i+1}P_{i}+beta_i P_{i-1}

```
> BETA0:=0;
```

$$BETA0 := 0$$

```
> N:=5;
```

$$N := 5$$

```
> BETA:=BETA0;
```

$$BETA := 0$$

```
> ALPHA0:=0;
```

$$ALPHA0 := 0$$

```
> ALPHA1:=ALPHA0;
```

$$ALPHA1 := 0$$

```
> P1 := x ->x-.5000000000;
```

$$P1 := x \to x - .5000000000$$

```
> for i from 1 to N do
> BETA0:=BETA0+(x[i]*(x[i]-0.5)):;
> ALPHA0:=ALPHA0+(x[i]*(x[i]-0.5)^2):
> ALPHA1:=ALPHA1+(x[i]-0.5)^2:
> od:;
> BETA1:=(1/N)*BETA0;
```

$$BETA1 := .1250000000$$

```
> ALPHA2:=ALPHA0/ALPHA1;
```

$$ALPHA2 := .5000000000$$

```
> P1:=x-0.5;
```

$$P1 := x - .5$$

```
> P2:=(x-ALPHA2)*P1-BETA1;
```

$$P2 := (x - .5000000000)(x - .5) - .1250000000$$

```
> P2 := x ->(x-0.5)*(x-0.5)-0.125;
```

$$P2 := x \to (x - .5)^2 - .125$$

OS POLINÔMIOS ORTOGONAIS SÃO: P0=1, P1 e P2

```
> b3:=0:;
> a33:=0:
> for i from 1 to 5 do
>
>
> a33:=a33+(P2(x[i]))^2;
> b3:=b3+f[i]*(P2(x[i]));
> od:;
>
>
> f33:=a33;
```

$$f33 := .05468750$$

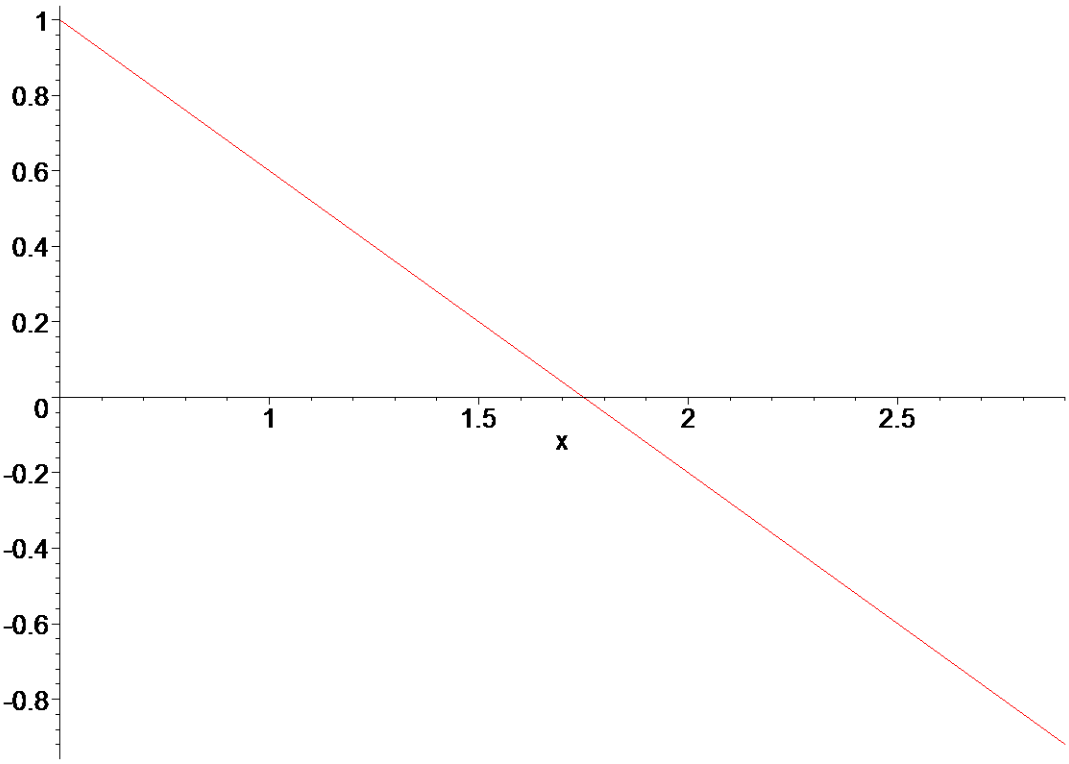
```
> a_2:=b3/a33;
```

$$a\_2 := 0$$

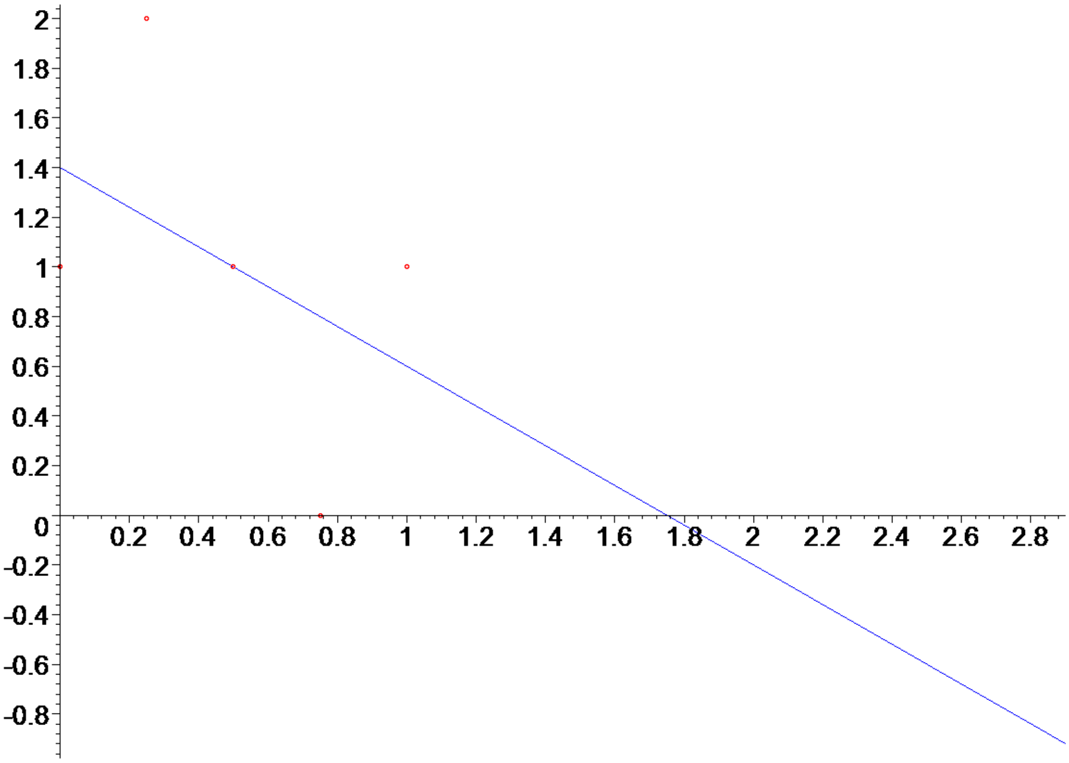
```
> Q2(x):=a_0+a_1*P_1+a_2*P_2;
```

$$Q2(x) := 1.400000000 - .8000000000\, x$$

```
>
>
> zb:=plot([Q2(x)], x=0.5..2.9, color=[blue], style=[line]):
>
> plot([Q2(x)], x=0.5..2.9, color=[red], style=[line]);
```




```
> display(plot([ZZ],style=[point],symbol=circle),z4,zb);
```


2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8