METODO DOS MÍNIMOS QUADRADOS

APROXIMAÇÃO POR RETAS, FUNÇÕES QUADRÁTICAS E CÚBICAS.

```
> restart:
> with(plots):
> with(linalg):
Warning, new definition for norm
Warning, new definition for trace
> x1[0]:=0:
> x2[0]:=0.5:
> x3[0]:=1.0:
> x4[0]:=1.5:
> x5[0]:=1.8:
> x6[0]:=2.1:
```

COLOCANDO OS PESOS W_i UNIFORMES

```
> w1[0]:=1:;
> w2[0]:=1:;
> w3[0]:=1:;
> w4[0]:=1:;
> w5[0]:=1:;
> w6[0]:=1:;
```

VALOR DA FUNÇÃO NOS PONTOS

```
> f1[0]:=5.0:
> f2[0]:=4.0:
> f3[0]:=4.0:
> f4[0]:=2.0:
> f5[0]:=2.5:
> f6[0]:=1.5:
```

MONTANDO O SISTEMA LINEAR PARA POLINÔMIOS DE GRAU 1,2 E 3

```
>
> X[0]:= array(1..6,[x1[0],x2[0],x3[0],x4[0],x5[0],x6[0]]):;
> a11:=w1[0]+w2[0]+w3[0]+w4[0]+w5[0]+w6[0]:;
> a12:=w1[0]*x1[0]+w2[0]*x2[0]+w3[0]*x3[0]+w4[0]*x4[0]+w5[0]*x5[0]+w
  6[0]*x6[0]:;
> a21:=a12:
> a22:=w1[0]*x1[0]^2+w2[0]*x2[0]^2+w3[0]*x3[0]^2+w4[0]*x4[0]^2+w5[0]
  *x5[0]^2+w6[0]*x6[0]^2:;
> a13:=a22:
> a31:=a13:
> a23:=w1[0]*x1[0]^3+w2[0]*x2[0]^3+w3[0]*x3[0]^3+w4[0]*x4[0]^3+w5[0]
  *x5[0]^3+w6[0]*x6[0]^3:;
> a33:=w1[0]*x1[0]^4+w2[0]*x2[0]^4+w3[0]*x3[0]^4+w4[0]*x4[0]^4+w5[0]
  *x5[0]^4+w6[0]*x6[0]^4:;
```

```
> a32:=a23:
> a14:=a23:
> a24:=a33:
> a34:=w1[0]*x1[0]^5+w2[0]*x2[0]^5+w3[0]*x3[0]^5+w4[0]*x4[0]^5+w5[0]
  *x5[0]^5+w6[0]*x6[0]^5:;
> a43:=a34:
> a44:=w1[0]*x1[0]^6+w2[0]*x2[0]^6+w3[0]*x3[0]^6+w4[0]*x4[0]^6+w5[0]
  *x5[0]^6+w6[0]*x6[0]^6:;
> a41:=a14:
> a42:=a24:
> b1:=w1[0]*f1[0]+w2[0]*f2[0]+w3[0]*f3[0]+w4[0]*f4[0]+w5[0]*f5[0]+w6
  [0]*f6[0]:;
> b2:=w1[0]*f1[0]*x1[0]+w2[0]*f2[0]*x2[0]+w3[0]*f3[0]*x3[0]+w4[0]*f4
  [0]*x4[0]+w5[0]*f5[0]*x5[0]+w6[0]*f6[0]*x6[0]:;
> b3:=w1[0]*f1[0]*x1[0]^2+w2[0]*f2[0]*x2[0]^2+w3[0]*f3[0]*x3[0]^2+w4
  [0]*f4[0]*x4[0]^2+w5[0]*f5[0]*x5[0]^2+w6[0]*f6[0]*x6[0]^2:;
>
> b4:=w1[0]*f1[0]*x1[0]^3+w2[0]*f2[0]*x2[0]^3+w3[0]*f3[0]*x3[0]^3+w4
  [0]*f4[0]*x4[0]^3+w5[0]*f5[0]*x5[0]^3+w6[0]*f6[0]*x6[0]^3:;
```

REGRESSÃO LINEAR: APROXIMANDO OS PONTOS PELO POLINÔMIO DE GRAU 1.

```
> A1:=matrix(2,2,[[a11,a12],[a21,a22]]);
```

$$A1 := \begin{bmatrix} 6 & 6.9 \\ 6.9 & 11.15 \end{bmatrix}$$

```
> B1:=vector(2,[b1,b2]);
> det(A1);
```

$$B1 := [19.0, 16.65]$$

$$19.29$$

```
> C1:=linsolve(A1,B1);
```

$$C1 := [5.026697767, -1.617418349]$$

```
> c11:=C1[1];
```

$$c11 := 5.026697767$$

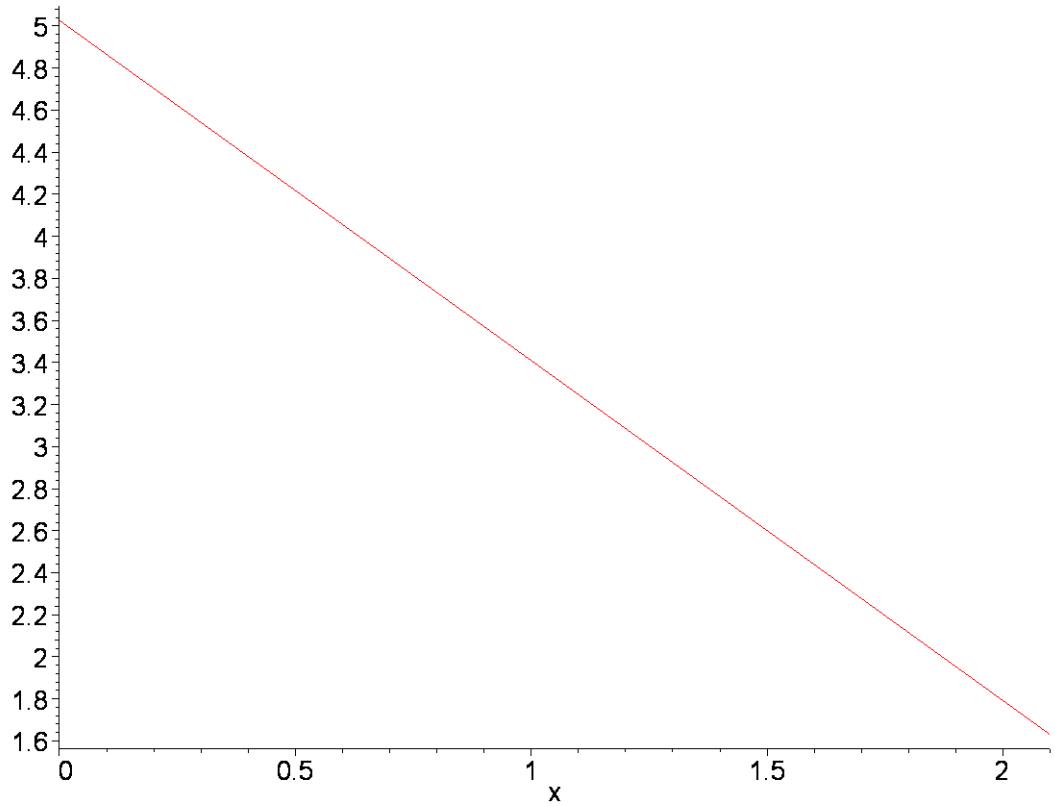
```
> c12:=C1[2];
```

$$c12 := -1.617418349$$

```
> phi1(x):=(C1[1]+C1[2]*x);
```

$$\phi 1(x) := 5.026697767 - 1.617418349\, x$$

```
>
> plot([phi1(x)], x=0..2.1, color=[red], style=[line]);
```




```
> N:=6;
> F:= vector([f1[0],f2[0], f3[0], f4[0],f5[0],f6[0]]);
```

$$N := 6$$

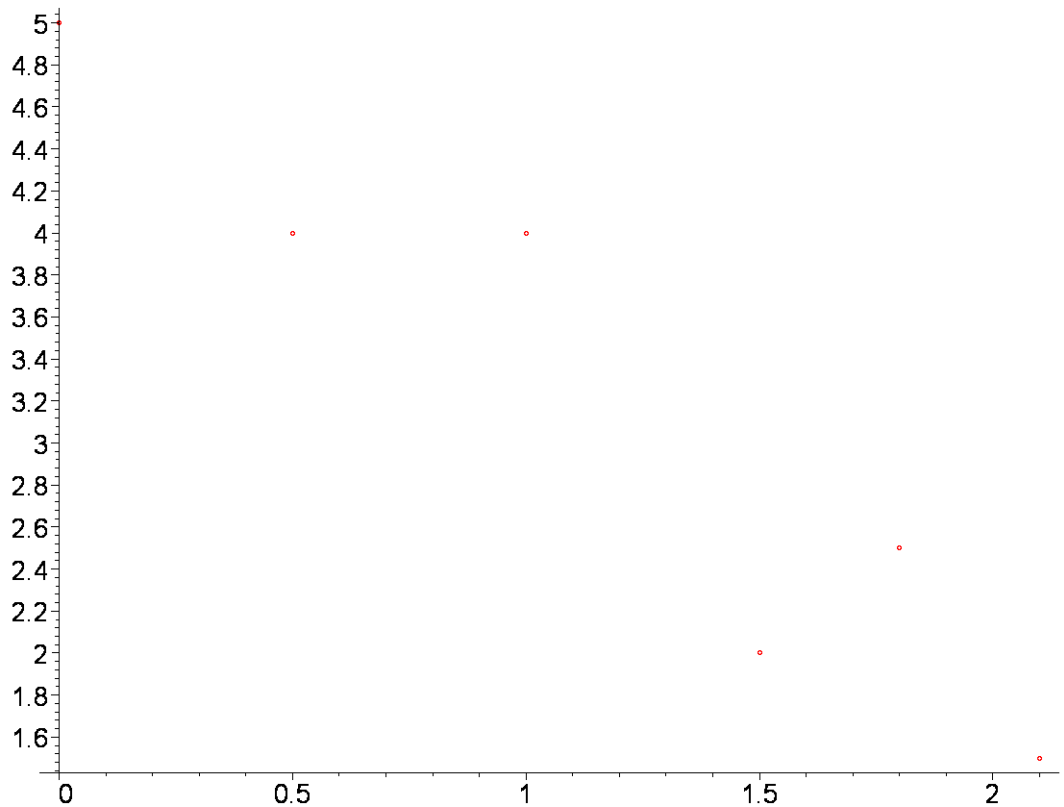
$$F := [5.0, 4.0, 4.0, 2.0, 2.5, 1.5]$$

```
> X:=vector([x1[0],x2[0],x3[0],x4[0],x5[0],x6[0]]);
>
```

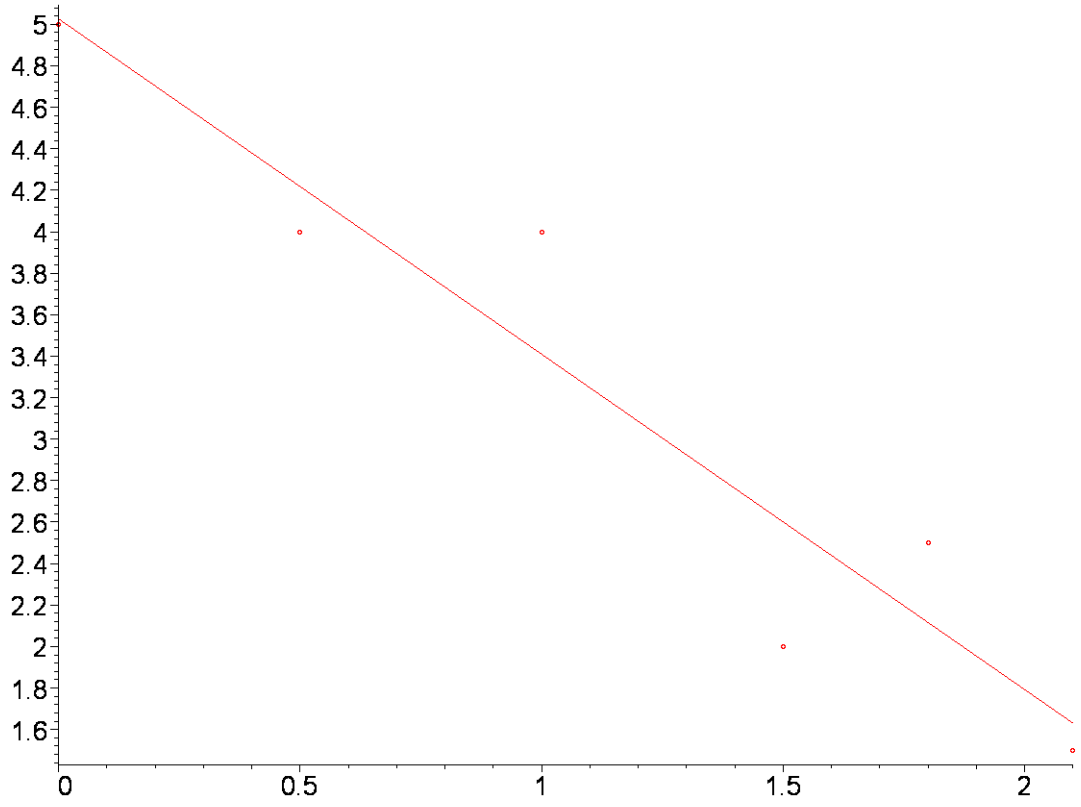
$$X := [0, .5, 1.0, 1.5, 1.8, 2.1]$$

```
> VRAP1:= [[ X[i], F[i]] $i=1..6];
  plot(VRAP1,style=point,symbol=circle);
```

$$VRAP1 := [[0, 5.0], [.5, 4.0], [1.0, 4.0], [1.5, 2.0], [1.8, 2.5], [2.1, 1.5]]$$




```
> gr1:=plot(phi1(x), x=0..2.1, color=[red],style=[line]):

> display(plot([VRAP1],style=[point],symbol=circle),gr1);
```




```
>
```

CALCULO DO ERRO: DERIVADO DOS MINIMOS QUADRADOS: NORMA EUCLIDIANA

```
>
>
```

POLINOMIOS DE GRAU 2.

```
> A2:=matrix(3,3,[[a11,a12,a13],[a21,a22,a23],[a31,a32,a33]]);
```

$$A2 := \begin{bmatrix} 6 & 6.9 & 11.15 \\ 6.9 & 11.15 & 19.593 \\ 11.15 & 19.593 & 36.0707 \end{bmatrix}$$

```
> B2:=vector(3,[b1,b2,b3]);
> det(A2);
```

$$B2 := [19.0, 16.65, 24.215]$$

$$21.068944$$

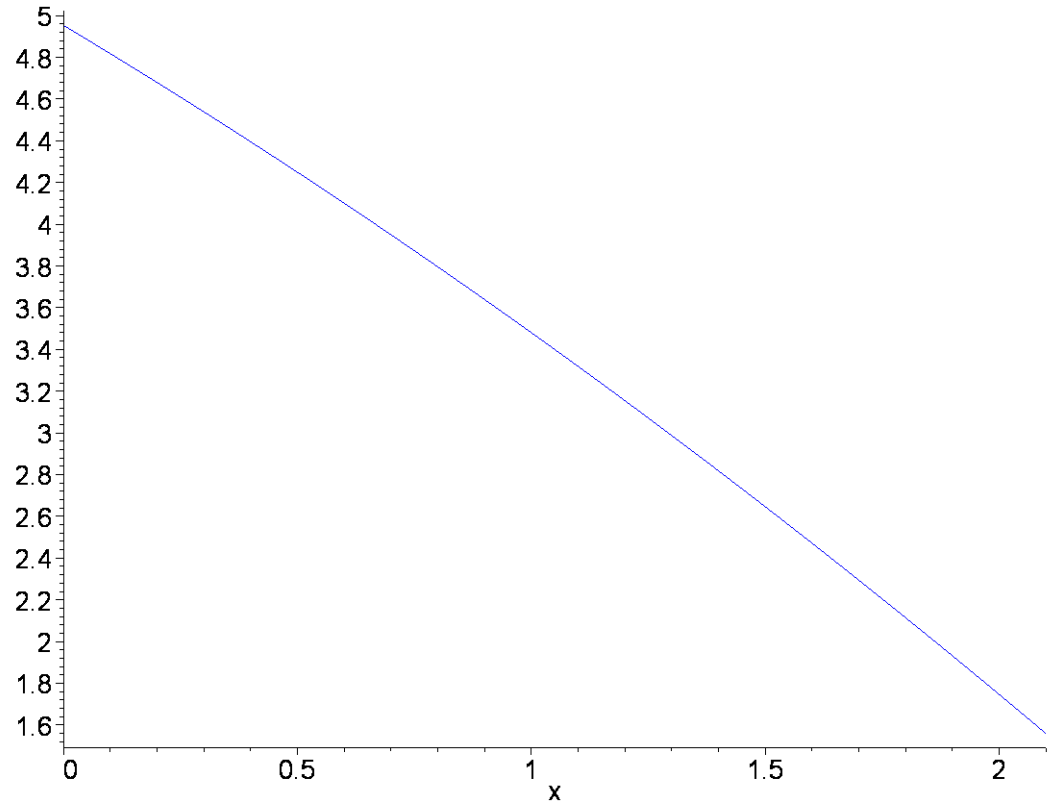
```
> C2:=linsolve(A2,B2);
```

$$C2 := [4.953131027, -1.342466928, -.1305618253]$$

```
> phi2(x):=(C2[1]+C2[2]*x+C2[3]*x^2);
```

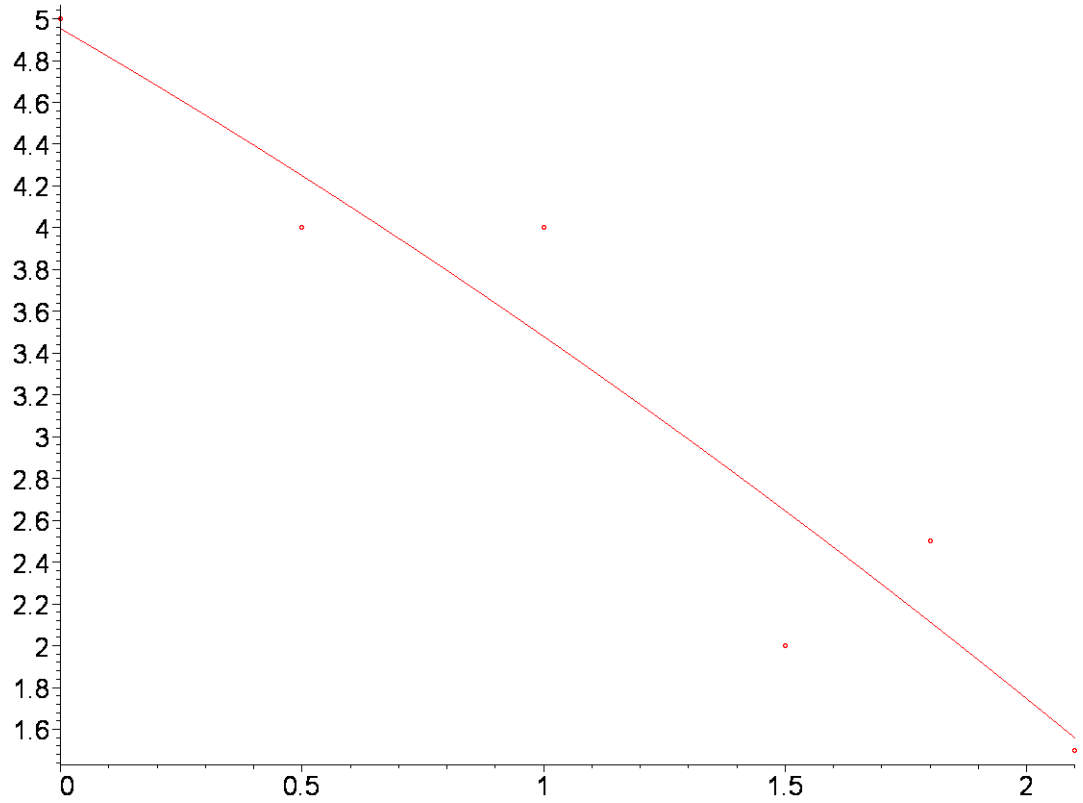
$$\phi 2(x) := 4.953131027 - 1.342466928\, x - .1305618253\, x^2$$

```
> plot([phi2(x)], x=0..2.1, color=[blue], style=[line]);
```




```
> gr2:=plot(phi2(x), x=0..2.1, color=[red],style=[line]):
```

```
>
> display(plot([VRAP1],style=[point],symbol=circle),gr2);
```




```
>
>
```

POLINOMIOS DE GRAU 3.

```
> A3:=matrix(4,4,[[a11,a12,a13,a14],[a21,a22,a23,a24],[a31,a32,a33,a
  34],[a41,a42,a43,a44]]);
```

$$A3 := \begin{bmatrix} 6 & 6.9 & 11.15 & 19.593 \\ 6.9 & 11.15 & 19.593 & 36.0707 \\ 11.15 & 19.593 & 36.0707 & 68.36169 \\ 19.593 & 36.0707 & 68.36169 & 132.184595 \end{bmatrix}$$

```
> B3:=vector(4,[b1,b2,b3,b4]);
> det(A3);
```

$$B3 := [19.0, 16.65, 24.215, 39.7215]$$

$$7.50166759$$

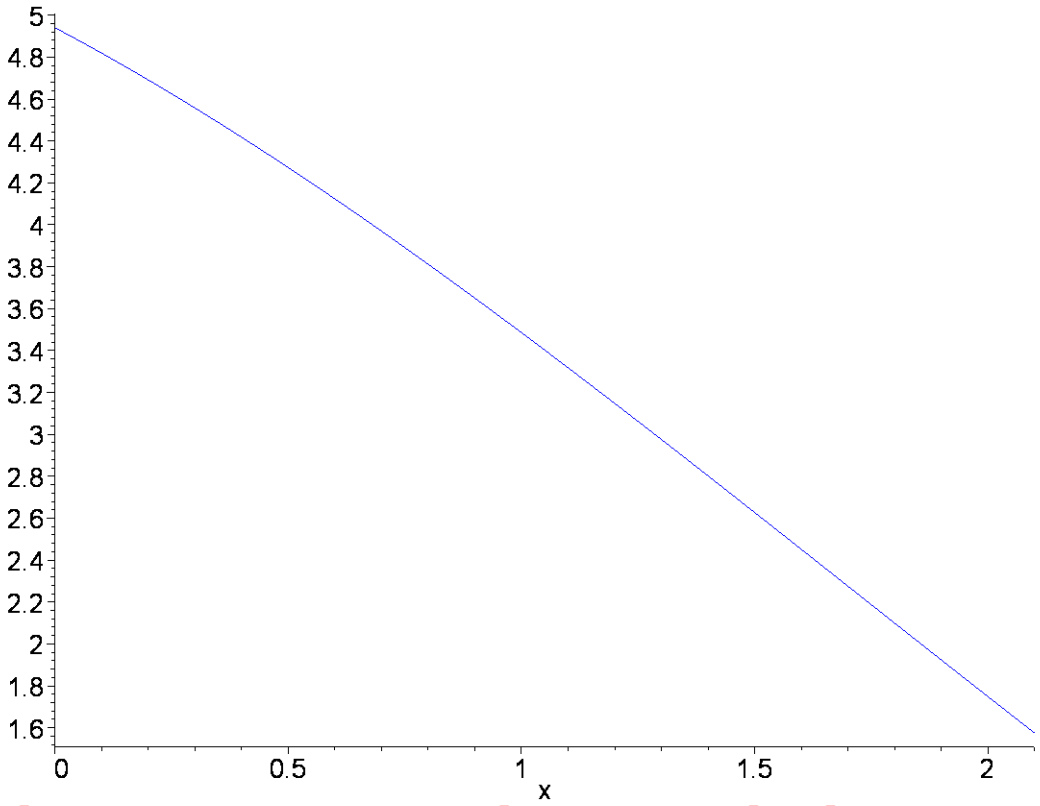
```
> C3:=linsolve(A3,B3);
```

$$C3 := [4.939854478, -1.179565811, -.3424394268, .0672728442]$$

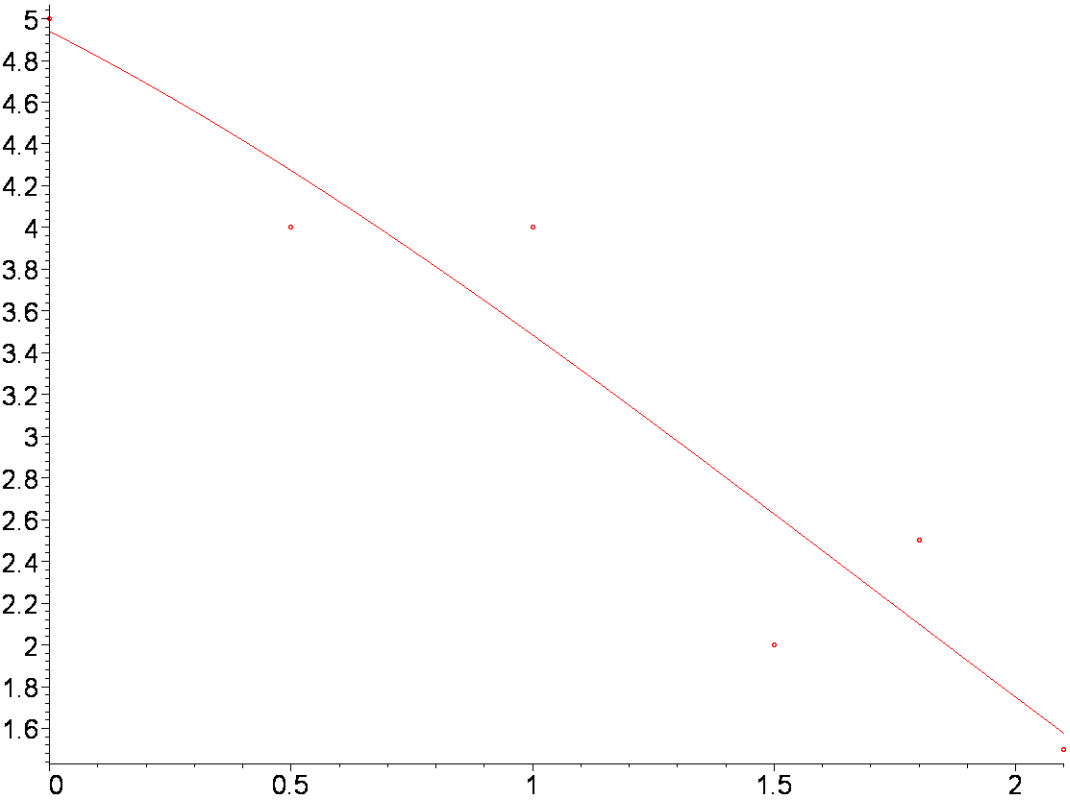
```
> phi3(x):=(C3[1]+C3[2]*x+C3[3]*x^2+C3[4]*x^3);
```

$$\phi 3(x) := 4.939854478 - 1.179565811\, x - .3424394268\, x^2 + .0672728442\, x^3$$

```
> plot([phi3(x)], x=0..2.1, color=[blue], style=[line]);
```

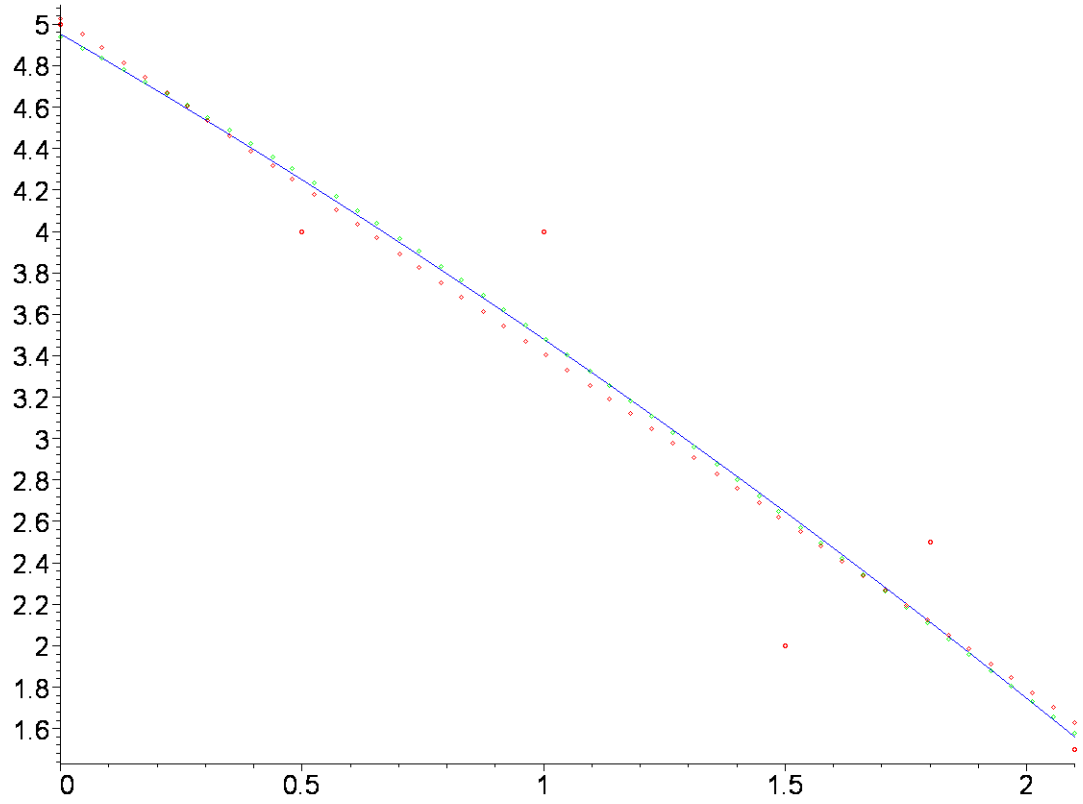



```
> gr3:=plot(phi3(x), x=0..2.1, color=[red],style=[line]):
> display(plot([VRAP1],style=[point],symbol=circle),gr3);
```




```
> gr1:=plot(phi1(x), x=0..2.1, color=[red],style=[point]):
```

```
> gr2:=plot(phi2(x), x=0..2.1, color=[blue],style=[line]):
> gr3:=plot(phi3(x), x=0..2.1, color=[green],style=[point]):
> display(plot([VRAP1],style=[point],symbol=circle),gr1,gr2,gr3);
```



USO DE PESOS DIFERENTES

PESOS

```
> w1[0]:=1:;
> w2[0]:=1:;
> w3[0]:=4:;
> w4[0]:=4:;
> w5[0]:=1:;
> w6[0]:=1:;
>
> X[0]:= array(1..6,[x1[0],x2[0],x3[0],x4[0],x5[0],x6[0]]):
Error, 1st index, 0, smaller than lower array bound 1
> a11:=w1[0]+w2[0]+w3[0]+w4[0]+w5[0]+w6[0]:;
> a12:=w1[0]*x1[0]+w2[0]*x2[0]+w3[0]*x3[0]+w4[0]*x4[0]+w5[0]*x5[0]+w
  6[0]*x6[0]:;
> a21:=a12:
> a22:=w1[0]*x1[0]^2+w2[0]*x2[0]^2+w3[0]*x3[0]^2+w4[0]*x4[0]^2+w5[0]
  *x5[0]^2+w6[0]*x6[0]^2:;
> a13:=a22:
> a31:=a13:
```

```
> a23:=w1[0]*x1[0]^3+w2[0]*x2[0]^3+w3[0]*x3[0]^3+w4[0]*x4[0]^3+w5[0]
  *x5[0]^3+w6[0]*x6[0]^3:;
> a33:=w1[0]*x1[0]^4+w2[0]*x2[0]^4+w3[0]*x3[0]^4+w4[0]*x4[0]^4+w5[0]
  *x5[0]^4+w6[0]*x6[0]^4:;
> a32:=a23:
> a14:=a23:
> a24:=a33:
> a34:=w1[0]*x1[0]^5+w2[0]*x2[0]^5+w3[0]*x3[0]^5+w4[0]*x4[0]^5+w5[0]
  *x5[0]^5+w6[0]*x6[0]^5:;
> a43:=a34:
> a44:=w1[0]*x1[0]^6+w2[0]*x2[0]^6+w3[0]*x3[0]^6+w4[0]*x4[0]^6+w5[0]
  *x5[0]^6+w6[0]*x6[0]^6:;
> a41:=a14:
> a42:=a24:
> b1:=w1[0]*f1[0]+w2[0]*f2[0]+w3[0]*f3[0]+w4[0]*f4[0]+w5[0]*f5[0]+w6
  [0]*f6[0]:;
> b2:=w1[0]*f1[0]*x1[0]+w2[0]*f2[0]*x2[0]+w3[0]*f3[0]*x3[0]+w4[0]*f4
  [0]*x4[0]+w5[0]*f5[0]*x5[0]+w6[0]*f6[0]*x6[0]:;
> b3:=w1[0]*f1[0]*x1[0]^2+w2[0]*f2[0]*x2[0]^2+w3[0]*f3[0]*x3[0]^2+w4
  [0]*f4[0]*x4[0]^2+w5[0]*f5[0]*x5[0]^2+w6[0]*f6[0]*x6[0]^2:;
>
> b4:=w1[0]*f1[0]*x1[0]^3+w2[0]*f2[0]*x2[0]^3+w3[0]*f3[0]*x3[0]^3+w4
  [0]*f4[0]*x4[0]^3+w5[0]*f5[0]*x5[0]^3+w6[0]*f6[0]*x6[0]^3:;
```

REGRESSÃO LINEAR: APROXIMANDO OS PONTOS PELO POLINÔMIO DE GRAU 1. COM PESOS

```
> A1:=matrix(2,2,[[a11,a12],[a21,a22]]);
```

$$A1 := \begin{bmatrix} 12 & 14.4 \\ 14.4 & 20.90 \end{bmatrix}$$

```
> B1:=vector(2,[b1,b2]);
> det(A1);
```

$$B1 := [37.0, 37.65]$$

$$43.44$$

```
> C1:=linsolve(A1,B1);
```

$$C1 := [5.320902391, -1.864640881]$$

```
> c11:=C1[1];
```

$$c11 := 5.320902391$$

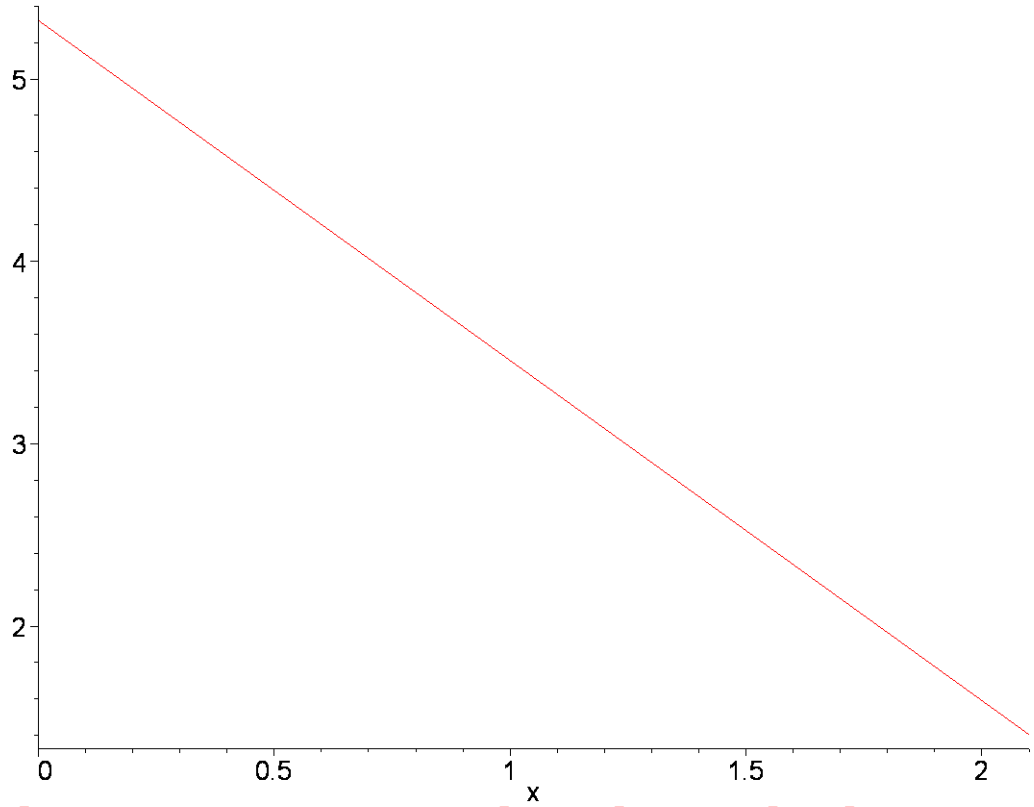
```
> c12:=C1[2];
```

$$c12 := -1.864640881$$

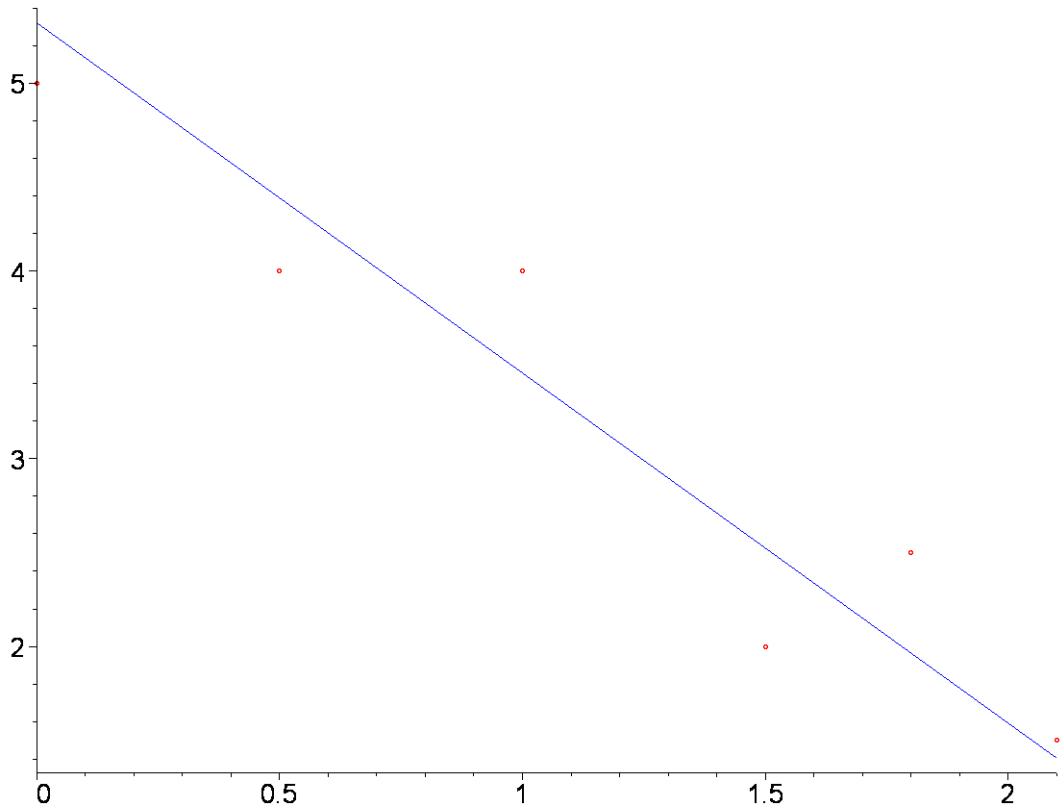
```
> phi1(x):=(C1[1]+C1[2]*x);
```

$$\phi 1(x) := 5.320902391 - 1.864640881\,x$$

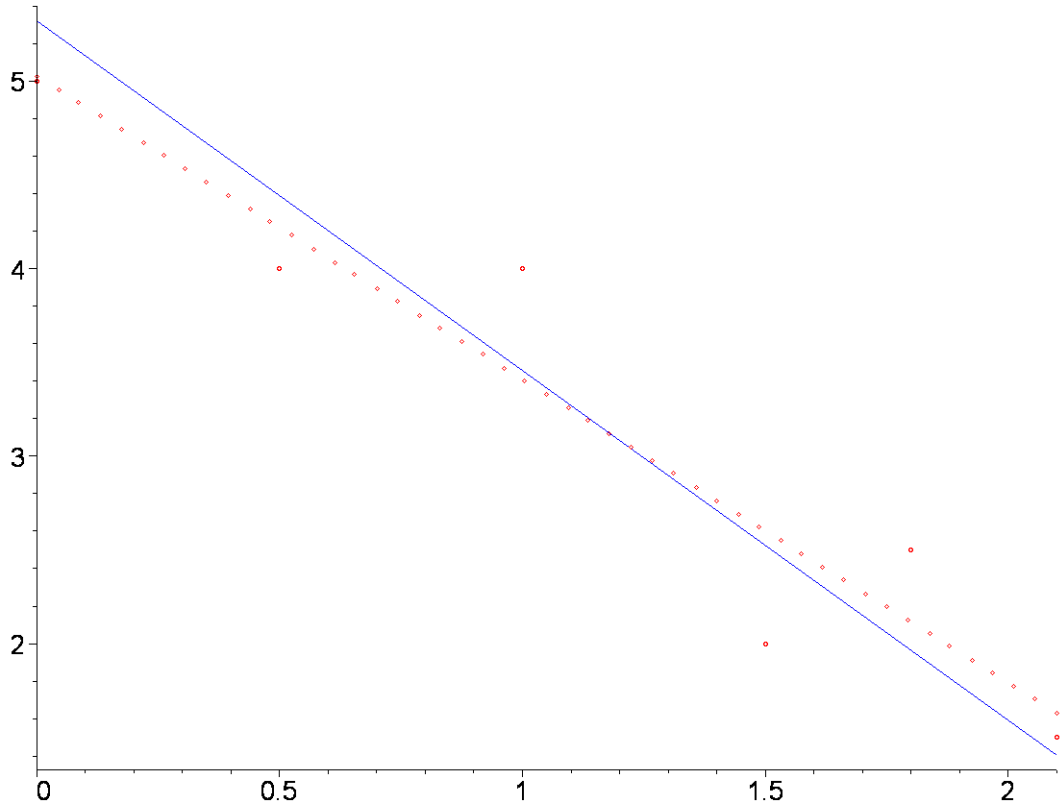
```
> 
> plot([phi1(x)], x=0..2.1, color=[red], style=[line]);
```




```
> gr4:=plot(phi1(x), x=0..2.1, color=[blue],style=[line]):
> 
> display(plot([VRAP1],style=[point],symbol=circle),gr4);
```




```
> display(plot([VRAP1],style=[point],symbol=circle),gr4,gr1);
```




```
>
```

```
> A2:=matrix(3,3,[[a11,a12,a13],[a21,a22,a23],[a31,a32,a33]]);
```

$$A2 := \begin{bmatrix} 12 & 14.4 & 20.90 \\ 14.4 & 20.90 & 32.718 \\ 20.90 & 32.718 & 54.2582 \end{bmatrix}$$

```
> B2:=vector(3,[b1,b2,b3]);
> det(A2);
```

$$B2 := [37.0, 37.65, 49.715]$$

$$75.655480$$

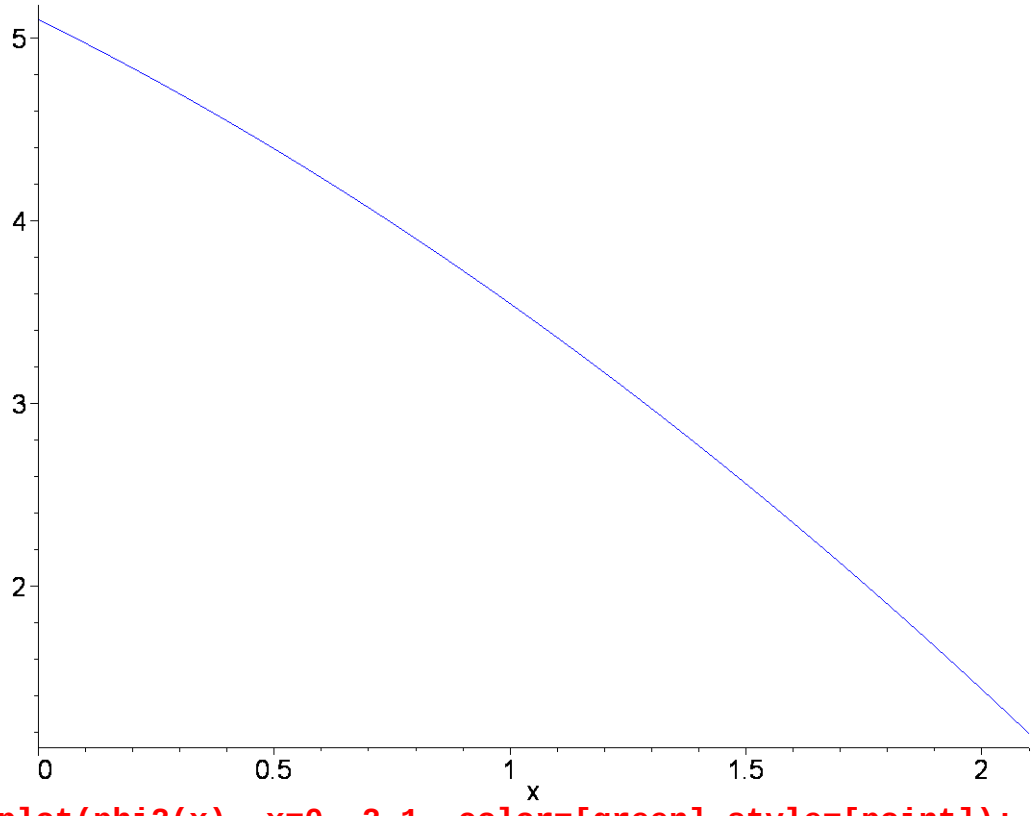
```
> C2:=linsolve(A2,B2);
```

$$C2 := [5.101039169, -1.277625064, -.2782138328]$$

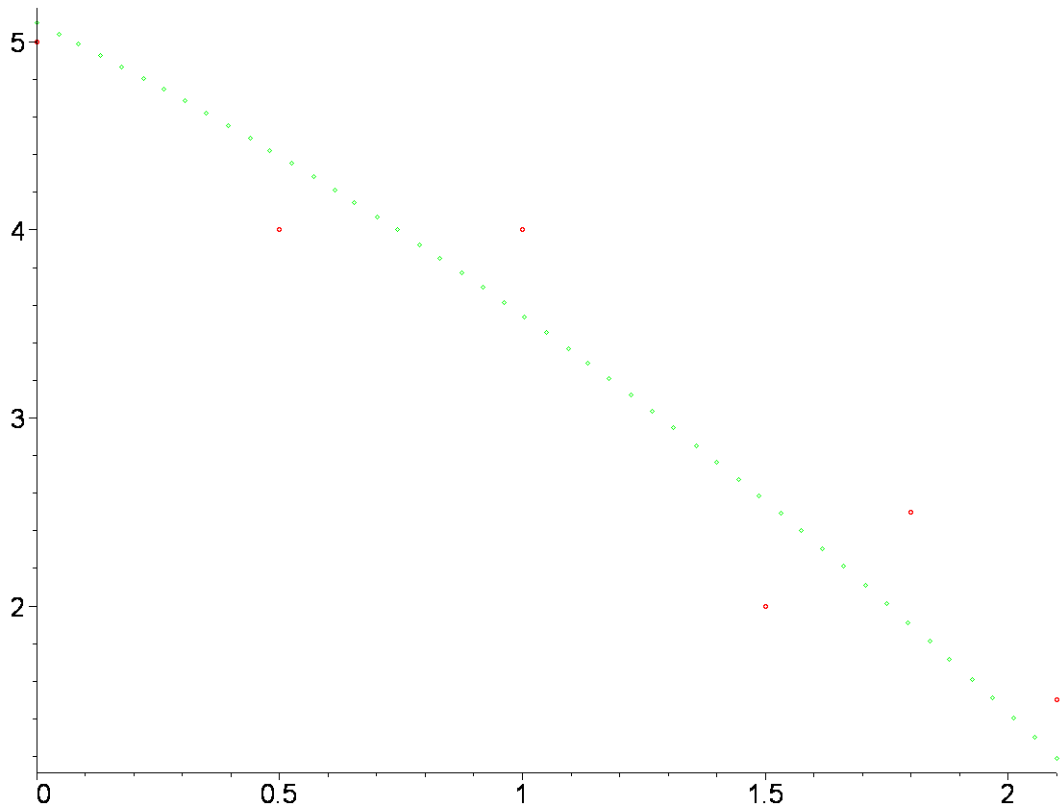
```
>

> phi2(x):=(C2[1]+C2[2]*x+C2[3]*x^2);
```

$$\phi 2(x) := 5.101039169 - 1.277625064\, x - .2782138328\, x^2$$

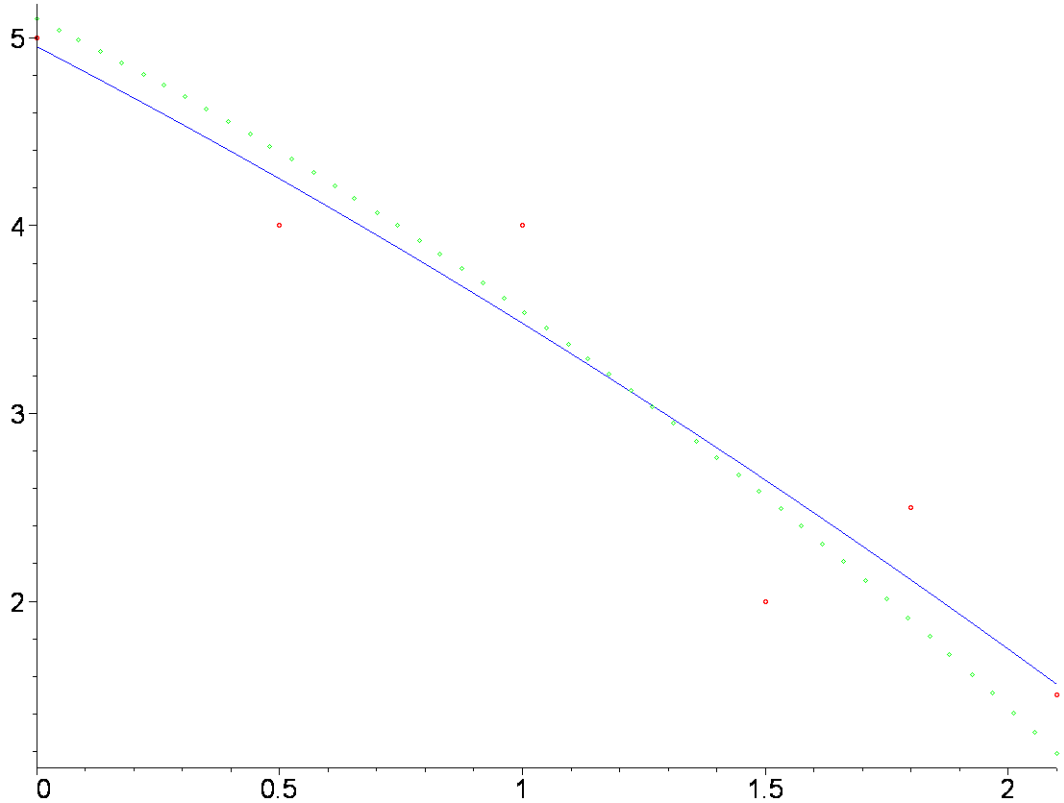
```
>
> plot([phi2(x)], x=0..2.1, color=[blue], style=[line]);
```




```
> gr5:=plot(phi2(x), x=0..2.1, color=[green],style=[point]):
> display(plot([VRAP1],style=[point],symbol=circle),gr5);
```




```
> display(plot([VRAP1],style=[point],symbol=circle),gr5,gr2);
```



```
>
>
> A3:=matrix(4,4,[[a11,a12,a13,a14],[a21,a22,a23,a24],[a31,a32,a33,a
  34],[a41,a42,a43,a44]]);
```

$$A3 := \begin{bmatrix} 12 & 14.4 & 20.90 & 32.718 \\ 14.4 & 20.90 & 32.718 & 54.2582 \\ 20.90 & 32.718 & 54.2582 & 94.14294 \\ 32.718 & 54.2582 & 94.14294 & 169.356470 \end{bmatrix}$$

```
> B3:=vector(4,[b1,b2,b3,b4]);
> det(A3);
```

$$B3 := [37.0, 37.65, 49.715, 71.9715]$$

$$41.3986315$$

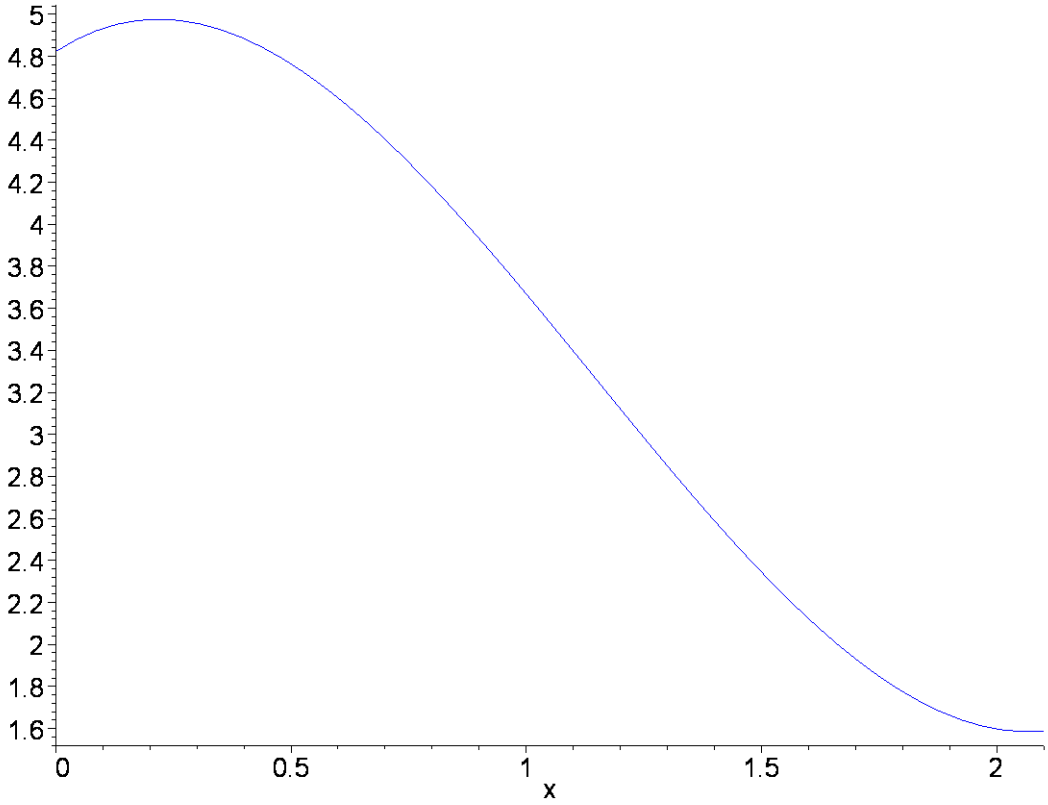
```
> C3:=linsolve(A3,B3);
```

$$C3 := [4.822510547, 1.458668943, -3.689221038, 1.076769984]$$

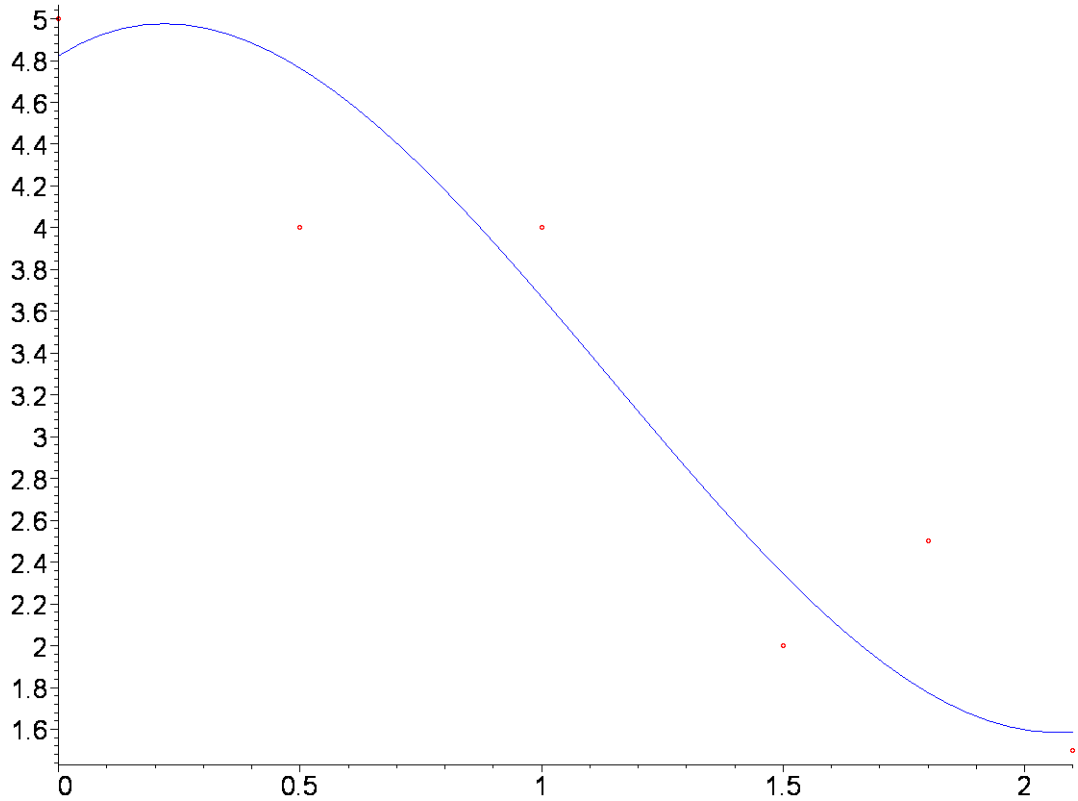
```
>
>
> phi3(x):=(C3[1]+C3[2]*x+C3[3]*x^2+C3[4]*x^3);
```

$$\phi 3(x) := 4.822510547 + 1.458668943\, x - 3.689221038\, x^2 + 1.076769984\, x^3$$

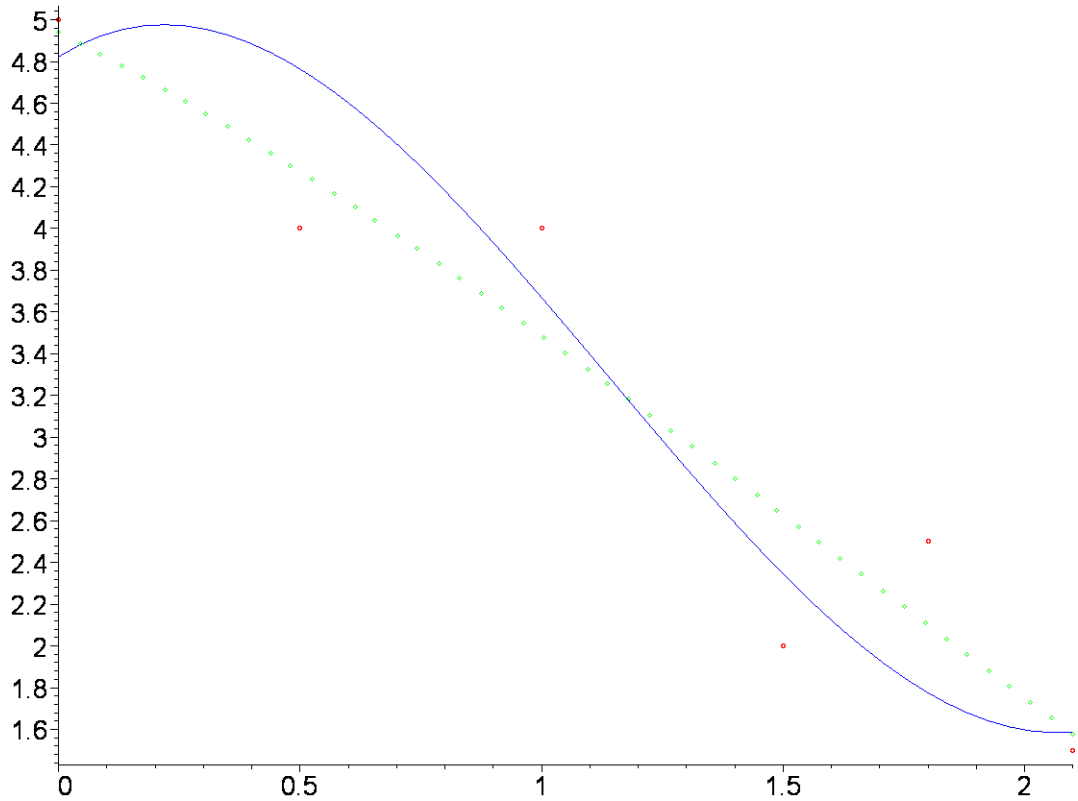
```
>
>
> plot([phi3(x)], x=0..2.1, color=[blue], style=[line]);
```



```
> gr6:=plot(phi3(x), x=0..2.1, color=[blue],style=[line]):
> display(plot([VRAP1],style=[point],symbol=circle),gr6);
```




```
> display(plot([VRAP1],style=[point],symbol=circle),gr6,gr3);
```




```
>
```

```
>
```
OUTRO EXEMPLO DE PESOS DIFERENTES
```
> w1[0]:=1:;
> w2[0]:=1:;
> w3[0]:=10:;
> w4[0]:=10:;
> w5[0]:=1:;
> w6[0]:=1:;
>
> X[0]:= array(1..6,[x1[0],x2[0],x3[0],x4[0],x5[0],x6[0]]):
Error, 1st index, 0, smaller than lower array bound 1
> a11:=w1[0]+w2[0]+w3[0]+w4[0]+w5[0]+w6[0]:;
> a12:=w1[0]*x1[0]+w2[0]*x2[0]+w3[0]*x3[0]+w4[0]*x4[0]+w5[0]*x5[0]+w
  6[0]*x6[0]:;
> a21:=a12:
> a22:=w1[0]*x1[0]^2+w2[0]*x2[0]^2+w3[0]*x3[0]^2+w4[0]*x4[0]^2+w5[0]
  *x5[0]^2+w6[0]*x6[0]^2:;
> a13:=a22:
> a31:=a13:
> a23:=w1[0]*x1[0]^3+w2[0]*x2[0]^3+w3[0]*x3[0]^3+w4[0]*x4[0]^3+w5[0]
  *x5[0]^3+w6[0]*x6[0]^3:;
> a33:=w1[0]*x1[0]^4+w2[0]*x2[0]^4+w3[0]*x3[0]^4+w4[0]*x4[0]^4+w5[0]
  *x5[0]^4+w6[0]*x6[0]^4:;
> a32:=a23:
> a14:=a23:
> a24:=a33:
> a34:=w1[0]*x1[0]^5+w2[0]*x2[0]^5+w3[0]*x3[0]^5+w4[0]*x4[0]^5+w5[0]
  *x5[0]^5+w6[0]*x6[0]^5:;
> a43:=a34:
> a44:=w1[0]*x1[0]^6+w2[0]*x2[0]^6+w3[0]*x3[0]^6+w4[0]*x4[0]^6+w5[0]
  *x5[0]^6+w6[0]*x6[0]^6:;
> a41:=a14:
> a42:=a24:
> b1:=w1[0]*f1[0]+w2[0]*f2[0]+w3[0]*f3[0]+w4[0]*f4[0]+w5[0]*f5[0]+w6
  [0]*f6[0]:;
> b2:=w1[0]*f1[0]*x1[0]+w2[0]*f2[0]*x2[0]+w3[0]*f3[0]*x3[0]+w4[0]*f4
  [0]*x4[0]+w5[0]*f5[0]*x5[0]+w6[0]*f6[0]*x6[0]:;
> b3:=w1[0]*f1[0]*x1[0]^2+w2[0]*f2[0]*x2[0]^2+w3[0]*f3[0]*x3[0]^2+w4
  [0]*f4[0]*x4[0]^2+w5[0]*f5[0]*x5[0]^2+w6[0]*f6[0]*x6[0]^2:;
>
> b4:=w1[0]*f1[0]*x1[0]^3+w2[0]*f2[0]*x2[0]^3+w3[0]*f3[0]*x3[0]^3+w4
  [0]*f4[0]*x4[0]^3+w5[0]*f5[0]*x5[0]^3+w6[0]*f6[0]*x6[0]^3:;
```

REGRESSÃO LINEAR: APROXIMANDO OS PONTOS PELO POLINÔMIO DE GRAU 1.

COM PESOS

```
> A1:=matrix(2,2,[[a11,a12],[a21,a22]]);
```

$$A1 := \begin{bmatrix} 24 & 29.4 \\ 29.4 & 40.40 \end{bmatrix}$$

```
> B1:=vector(2,[b1,b2]);
> det(A1);
```

$$B1 := [73.0, 79.65]$$

$$105.24$$

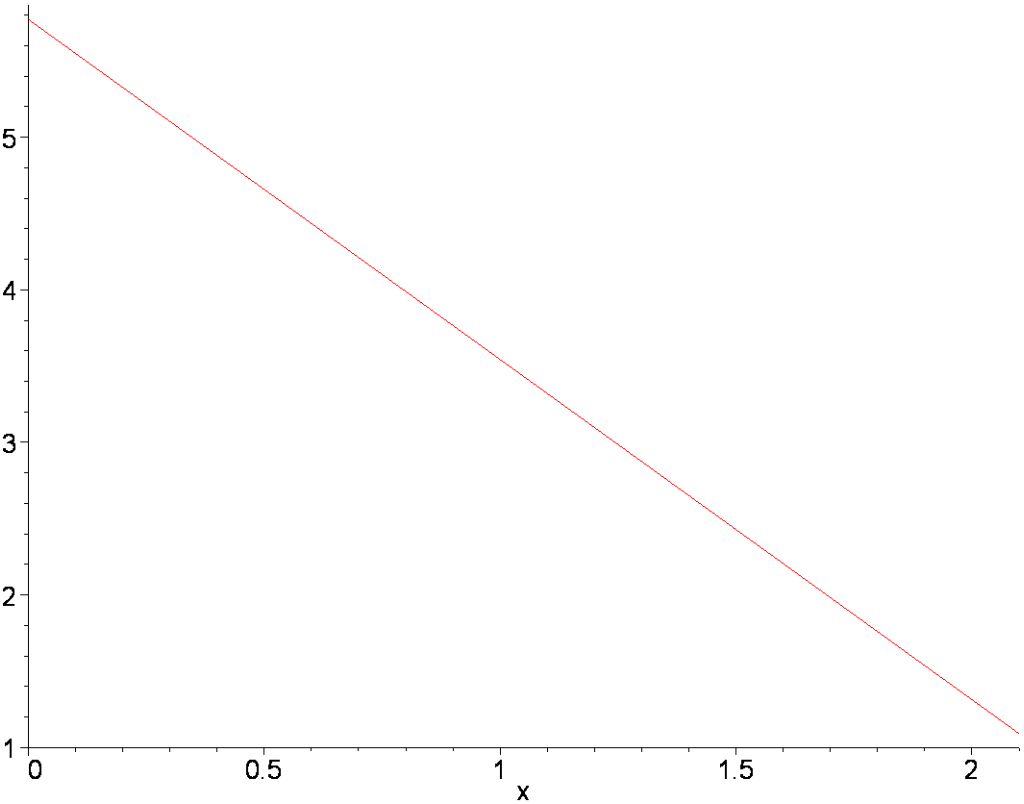
```
> C1:=linsolve(A1,B1);
```

$$C1 := [5.772424937, -2.229190425]$$

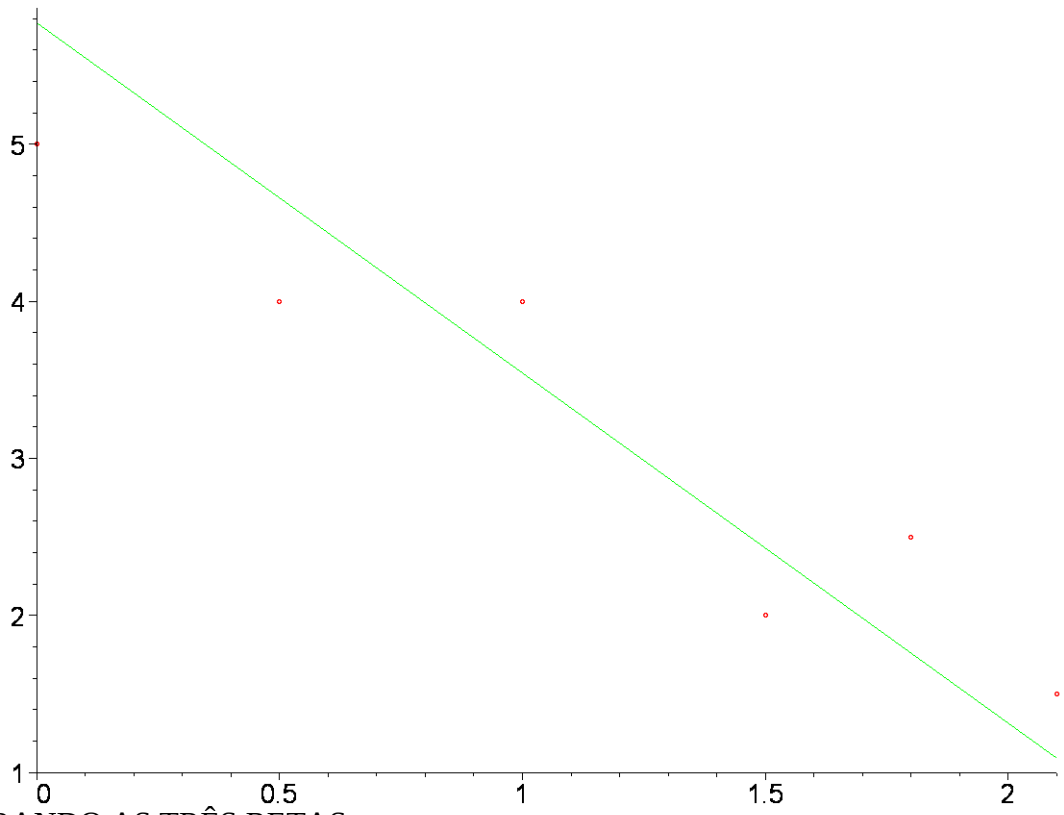
```
>
> phi1(x):=(C1[1]+C1[2]*x);
```

$$\phi 1(x) := 5.772424937 - 2.229190425\, x$$

```
>
> plot([phi1(x)], x=0..2.1, color=[red], style=[line]);
```

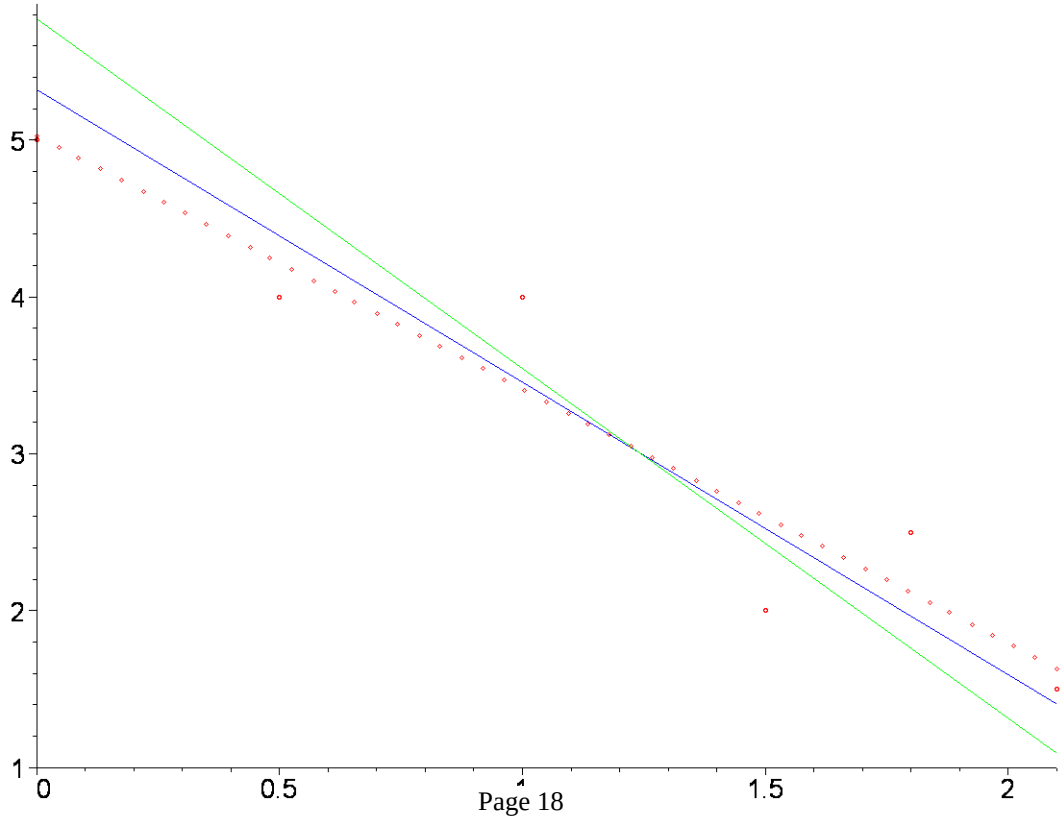



```
> gr8:=plot(phi1(x), x=0..2.1, color=[green],style=[line]):
> display(plot([VRAP1],style=[point],symbol=circle),gr8);
```



COMPARANDO AS TRÊS RETAS

```
> display(plot([VRAP1],style=[point],symbol=circle),gr8,gr4,gr1);
```



```
>
```

POLINOMIOS DE GRAU 2. COM PESOS

```
> A2:=matrix(3,3,[[a11,a12,a13],[a21,a22,a23],[a31,a32,a33]]);
```

$$A2 := \begin{bmatrix} 24 & 29.4 & 40.40 \\ 29.4 & 40.40 & 58.968 \\ 40.40 & 58.968 & 90.6332 \end{bmatrix}$$

```
> B2:=vector(3,[b1,b2,b3]);
> det(A2);
```

$$B2 := [73.0, 79.65, 100.715]$$

$$225.236752$$

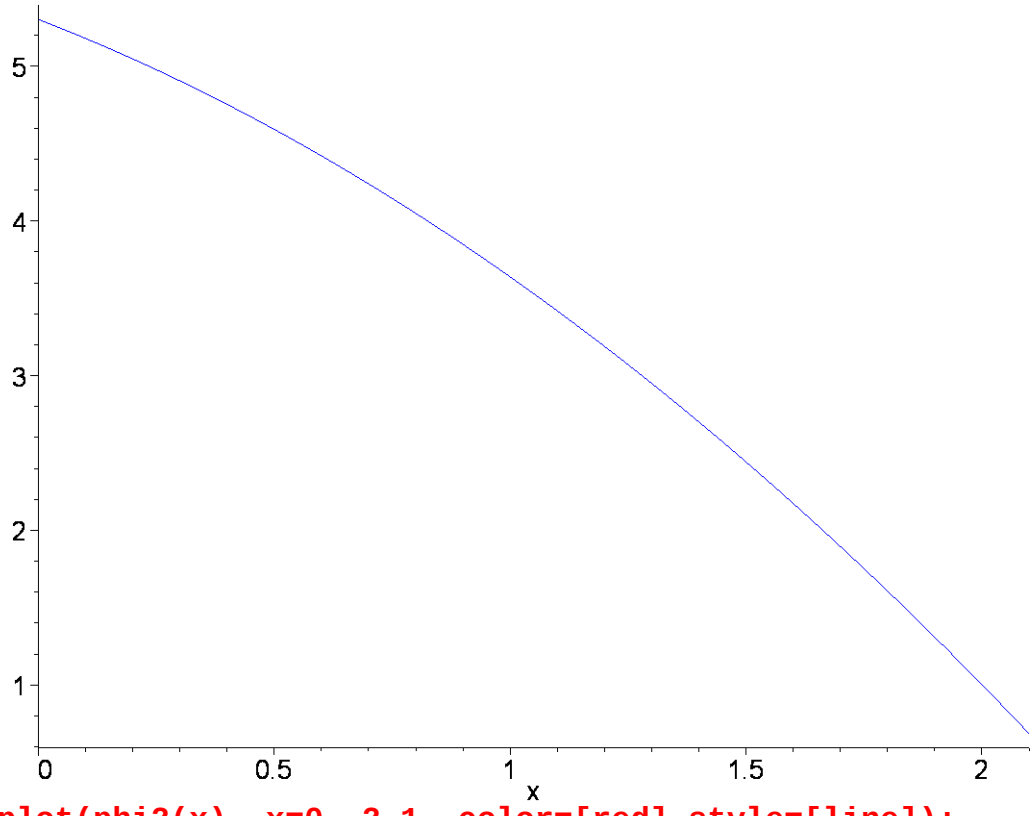
```
> C2:=linsolve(A2,B2);
```

$$C2 := [5.303736176, -1.178802172, -.485962485]$$

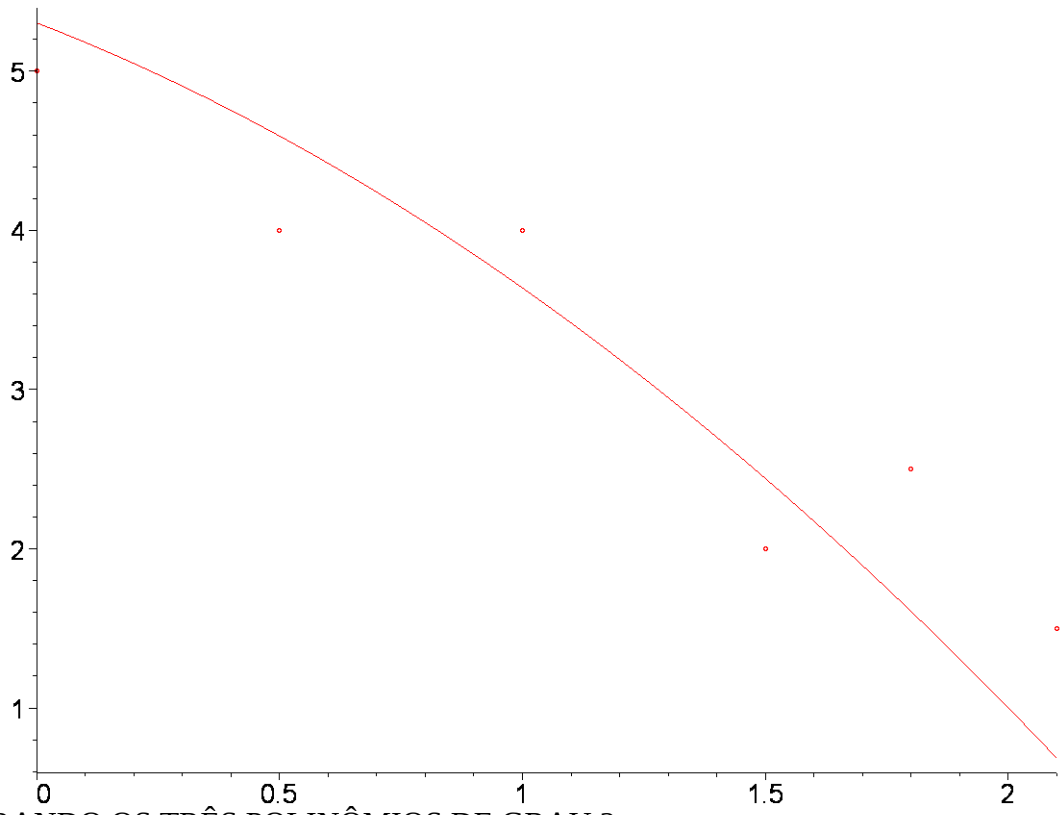
```
> phi2(x):=(C2[1]+C2[2]*x+C2[3]*x^2);
```

$$\phi 2(x) := 5.303736176 - 1.178802172\, x - .485962485\, x^2$$

```
> plot([phi2(x)], x=0..2.1, color=[blue], style=[line]);
```

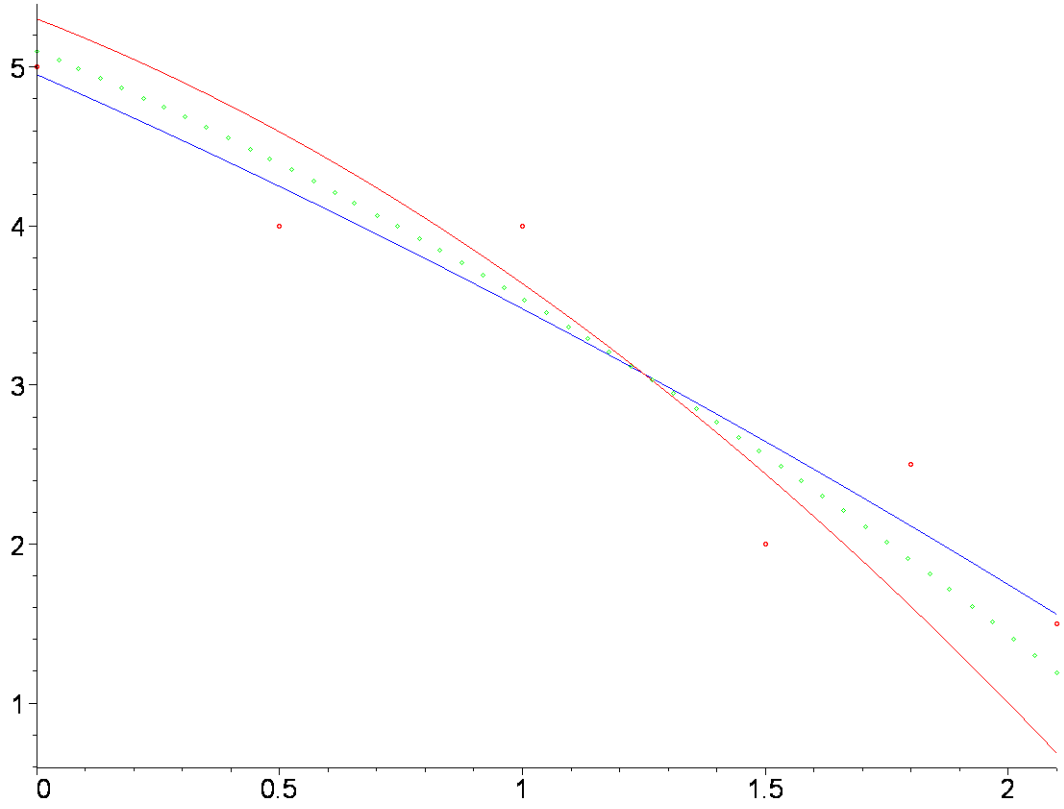



```
> gr9:=plot(phi2(x), x=0..2.1, color=[red],style=[line]):
> display(plot([VRAP1],style=[point],symbol=circle),gr9);
```



COMPARANDO OS TRÊS POLINÔMIOS DE GRAU 2

```
> display(plot([VRAP1],style=[point],symbol=circle),gr9,gr5,gr2);
```



POLINOMIOS DE GRAU 3. COM PESOS

```
>
> A3:=matrix(4,4,[[a11,a12,a13,a14],[a21,a22,a23,a24],[a31,a32,a33,a
  34],[a41,a42,a43,a44]]);
```

$$A3 := \begin{bmatrix} 24 & 29.4 & 40.40 & 58.968 \\ 29.4 & 40.40 & 58.968 & 90.6332 \\ 40.40 & 58.968 & 90.6332 & 145.70544 \\ 58.968 & 90.6332 & 145.70544 & 243.700220 \end{bmatrix}$$

```
> B3:=vector(4,[b1,b2,b3,b4]);
> det(A3);
```

$$B3 := [73.0, 79.65, 100.715, 136.4715]$$

$$179.232360$$

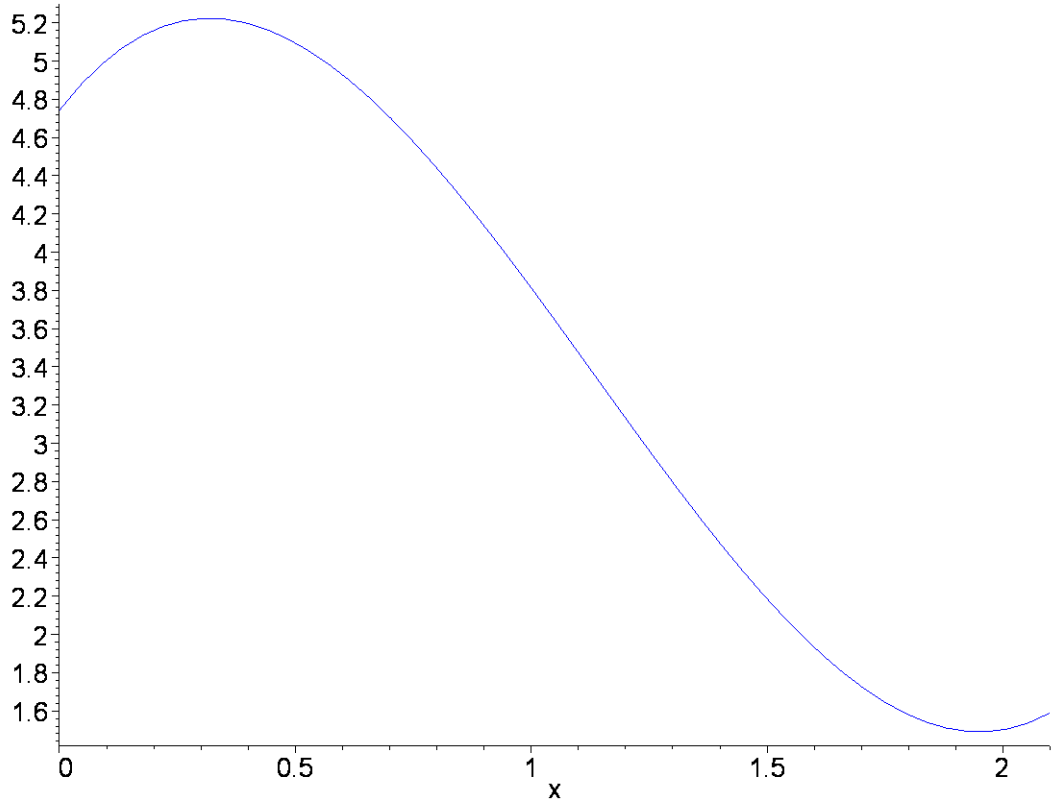
```
> C3:=linsolve(A3,B3);
```

$$C3 := [4.742877616, 3.197875090, -5.844797228, 1.717596283]$$

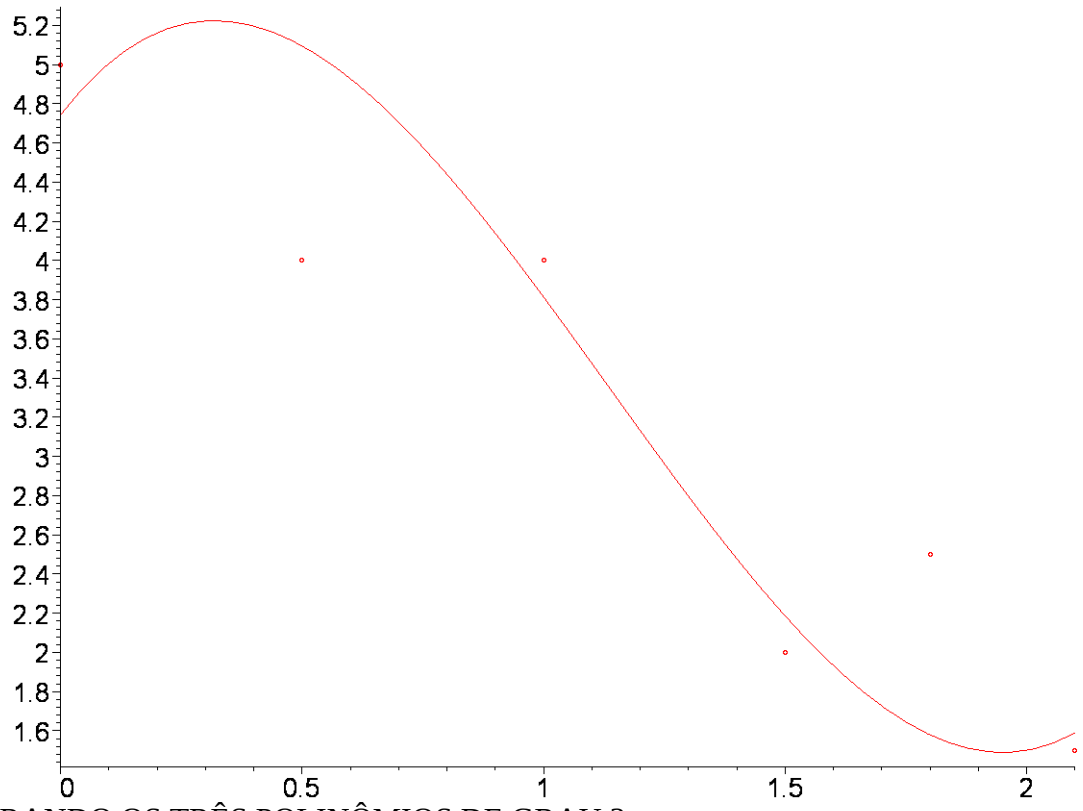
```
> phi3(x):=(C3[1]+C3[2]*x+C3[3]*x^2+C3[4]*x^3);
```

$$\phi 3(x) := 4.742877616 + 3.197875090\, x - 5.844797228\, x^2 + 1.717596283\, x^3$$

```
> plot([phi3(x)], x=0..2.1, color=[blue], style=[line]);
```



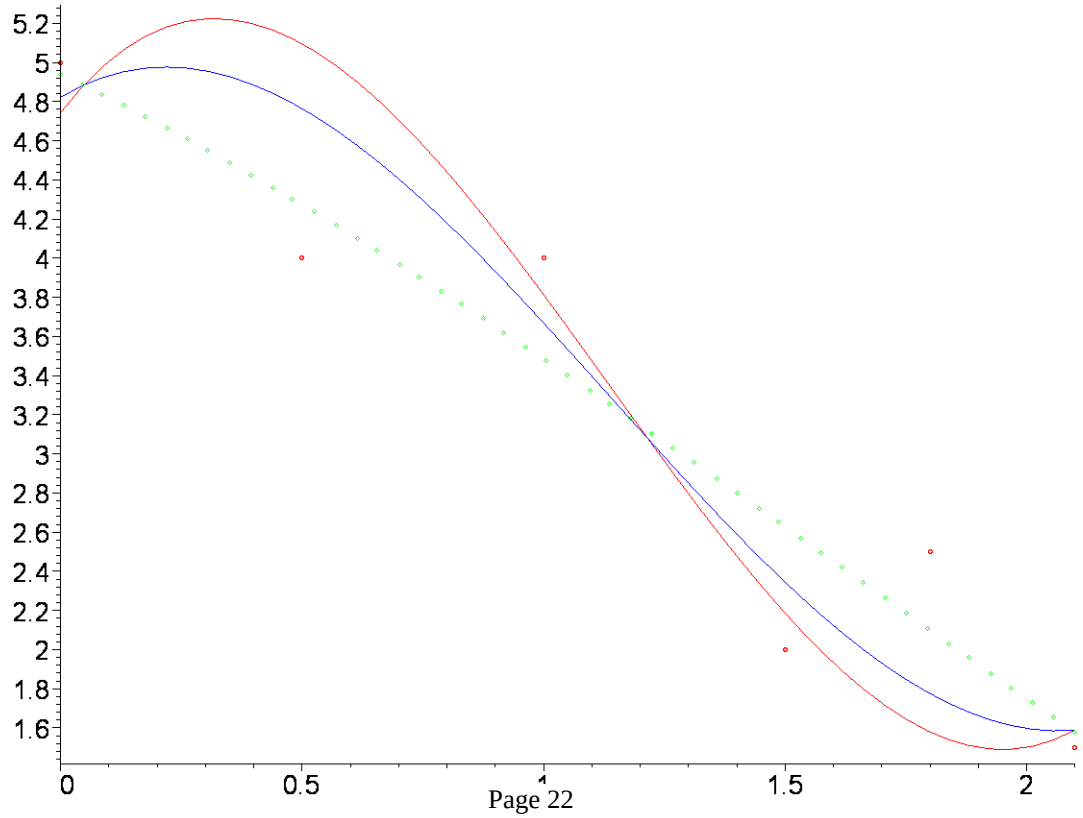

```
> gr10:=plot(phi3(x), x=0..2.1, color=[red],style=[line]):
> display(plot([VRAP1],style=[point],symbol=circle),gr10);
```



COMPARANDO OS TRÊS POLINÔMIOS DE GRAU 3.

Observe que quanto mais aumentamos o pesos nos pontos 3 e 4 mais a curva se aproxima desses valores.

```
> display(plot([VRAP1],style=[point],symbol=circle),gr6,gr10,gr3);
```



[>