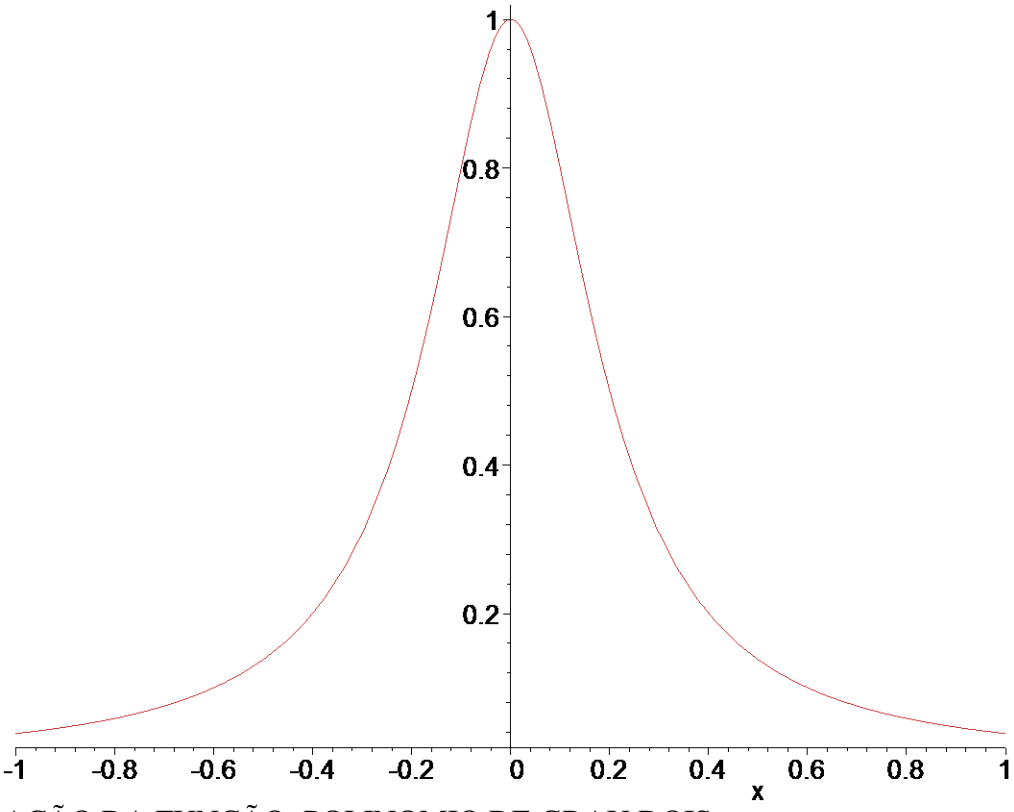
```
> restart:
> with(plots):
> with(linalg):
Warning, new definition for norm
Warning, new definition for trace
>
```

FENÔMENO DE RUNGE

```
> f:=x->(1/(1+25*x^2));
```

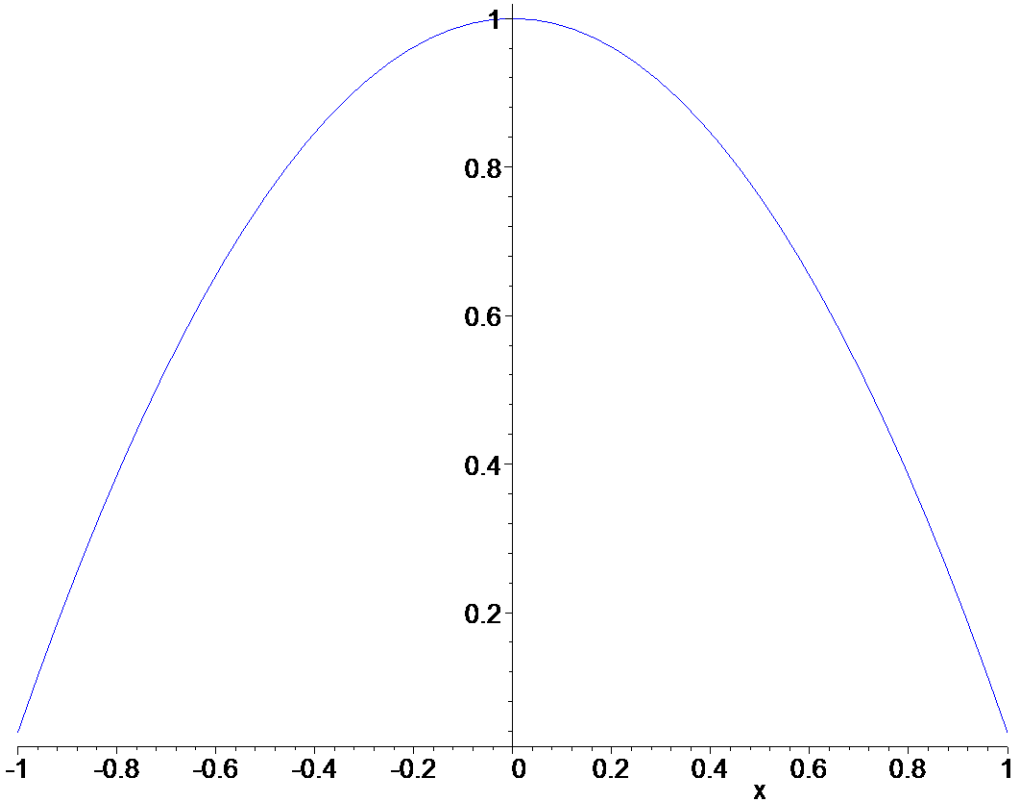
$$f := x \to \frac{1}{1 + 25\,x^2}$$

```
> z:=plot([f(x)], x=-1.0..1.0, color=[blue], style=[line]):
> plot([f(x)], x=-1.0..1.0, color=[orange], style=[line]);
```



INTERPOLAÇÃO DA FUNÇÃO: POLINOMIO DE GRAU DOIS

```
> x0:=-1:
> x1:=0.0:
> x2:=1.0:
> y0:=f(x0):;
> y1:=f(x1):;
> y2:=f(x2):;
```

```
> h:=1.0:
```
OPERADOR DIFERENÇAS DIVIDIDAS DE ORDEM 1
```
> f[x0,x1]:=(y1-y0)/h;
```
$$f_{-1,\,0} := .9615384615$$
```
> f[x1,x2]:=(y2-y1)/h;
```
$$f_{0,\,1.0} := -.9615384615$$
OPERADOR DIFERENÇAS DIVIDIDAS DE ORDEM 2
```
> f[x0,x1,x2]:=(f[x1,x2]-f[x0,x1])/(2*h);
```
$$f_{-1,\,0,\,1.0} := -.9615384615$$
```
> P2:=y0+f[x0,x1]*(x-x0)+f[x0,x1,x2]*(x-x0)*(x-x1);
```
$$P2 := 1.000000000 + .9615384615\,x - .9615384615\,(x+1)\,x$$
POLINOMIO INTERPOLADOR DE GRAU 2
```
> Runge2:=plot([P2(x)], x=-1.0..1.0, color=[blue], style=[line]):
> plot([P2(x)], x=-1.0..1.0, color=[blue], style=[line]);
```


```
>
> with(linalg):
>
> with(plots):
```
INTERPOLAÇÃO DA FUNÇÃO: POLINOMIO DE GRAU QUATRO
```
> f:=x->(1/(1+25*x^2)):
```
PONTOS CONHECIDOS
```
> x0:=-1:
```

```
> x1:=-0.5:
> x2:=0.0:
> x3:=0.5:
> x4:=1.0:
>
```

VALOR DA FUNÇÃO

```
> y0:=f(x0):;
> y1:=f(x1):
> y2:=f(x2):;
> y3:=f(x3):
> y4:=f(x4):
> h:=0.5:
```

OPERADOR DIFERENÇAS DIVIDIDAS DE ORDEM 1

```
> f[x0,x1]:=(y1-y0)/h;
```

$$f_{-1,\ -.5} := .1989389921$$

```
> f[x1,x2]:=(y2-y1)/h;
```

$$f_{-.5,\ 0} := 1.724137931$$

```
> f[x2,x3]:=(y3-y2)/h;
```

$$f_{0,\ .5} := -1.724137931$$

```
> f[x3,x4]:=(y4-y3)/h;
```

$$f_{.5,\ 1.0} := -.1989389921$$

OPERADOR DIFERENÇAS DIVIDIDAS DE ORDEM 2

```
> f[x0,x1,x2]:=(f[x1,x2]-f[x0,x1])/(2*h);
```

$$f_{-1,\ -.5,\ 0} := 1.525198939$$

```
> f[x1,x2,x3]:=(f[x2,x3]-f[x1,x2])/(2*h);
```

$$f_{-.5,\ 0,\ .5} := -3.448275862$$

```
> f[x2,x3,x4]:=(f[x3,x4]-f[x2,x3])/(2*h);
```

$$f_{0,\ .5,\ 1.0} := 1.525198939$$

OPERADOR DIFERENÇAS DIVIDIDAS DE ORDEM 3

```
> f[x0,x1,x2,x3]:=(f[x1,x2,x3]-f[x0,x1,x2])/(3*h);
> f[x1,x2,x3,x4]:=(f[x2,x3,x4]-f[x1,x2,x3])/(3*h);
```

$$f_{-1,\ -.5,\ 0,\ .5} := -3.315649867$$

$$f_{-.5,\ 0,\ .5,\ 1.0} := 3.315649867$$

OPERADOR DIFERENÇAS DIVIDIDAS DE ORDEM 4

```
> f[x0,x1,x2,x3,x4]:=(f[x1,x2,x3,x4]-f[x0,x1,x2,x3])/(4*h);
```

$$f_{-1,\ -.5,\ 0,\ .5,\ 1.0} := 3.315649868$$

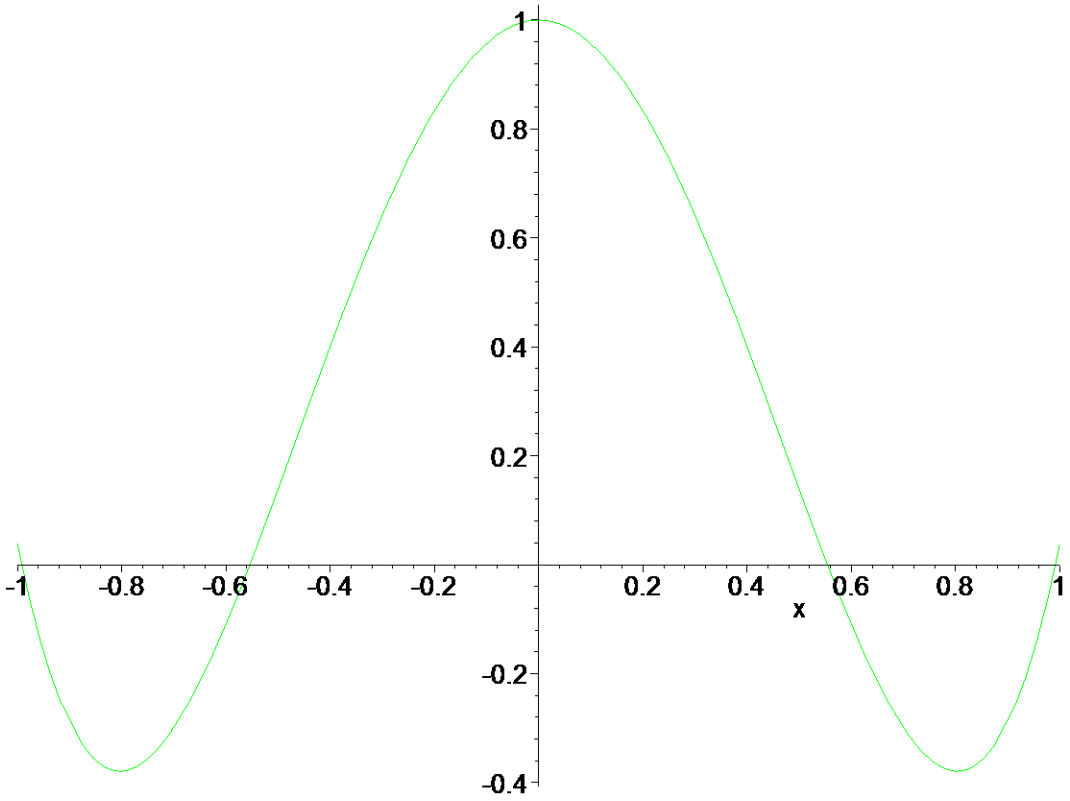
POLINOMIO INTERPOLADOR DE GRAU 4

```
> P4:=y0+f[x0,x1]*(x-x0)+f[x0,x1,x2]*(x-x0)*(x-x1)+f[x0,x1,x2,x3]*(x
```

```
   -x0)*(x-x1)*(x-x2)+f[x0,x1,x2,x3,x4]*(x-x0)*(x-x1)*(x-x2)*(x-x3);
>
>
```

$$P4 := .2374005306 + .1989389921\,x + 1.525198939\,(x+1)\,(x+.5) - 3.315649867\,(x+1)\,(x+.5)\,x + 3.315649868\,(x+1)\,(x+.5)\,x\,(x-.5)$$

```
>
> Runge4:=plot([P4(x)], x=-1.0..1.0, color=[blue], style=[line]):
> plot([P4(x)], x=-1.0..1.0, color=[green], style=[line]);
```



POINTERPOLAÇÃO DA FUNÇÃO: POLINOMIO DE GRAU DEZ
PONTOS CONHECIDOS

```
> with(linalg):
>
> with(plots):
> f:=x->(1/(1+25*x^2)):
>
> x0:=-1:
> x1:=-0.8:
> x2:=-0.6:
> x3:=-0.4:
> x4:=-0.2:
> x5:=0.0:
```

```
> x6:=0.2:
> x7:=0.4:
> x8:=0.6:
> x9:=0.8:
> x10:=1.0:
```

VALOR DA FUNÇÃO

```
> y0:=f(x0):
> y1:=f(x1):
> y2:=f(x2):
> y3:=f(x3):
> y4:=f(x4):
> y5:=f(x5):
> y6:=f(x6):
> y7:=f(x7):
> y8:=f(x8):
> y9:=f(x9):
> y10:=f(x10):
> h:=0.2:
>
> X[0]:= array(1..11,[x0,x1,x2,x3,x4,x5,x6,x7,x8,x9,x10]);
```

$$X_0 := [-1, -.8, -.6, -.4, -.2, 0, .2, .4, .6, .8, 1.0]$$

TABELA DAS DIFERENÇAS DIVIDIDAS: ODEM 1

```
>
> f[x0,x1]:=(y1-y0)/h;
```

$$f_{-1,\,-.8} := .1018099548$$

```
> f[x1,x2]:=(y2-y1)/h;
```

$$f_{-.8,\,-.6} := .2058823530$$

```
> f[x2,x3]:=(y3-y2)/h;
```

$$f_{-.6,\,-.4} := .5000000000$$

```
> f[x3,x4]:=(y4-y3)/h;
```

$$f_{-.4,\,-.2} := 1.500000000$$

```
> f[x4,x5]:=(y5-y4)/h;
```

$$f_{-.2,\,0} := 2.500000000$$

```
> f[x5,x6]:=(y6-y5)/h;
```

$$f_{0,\,.2} := -2.500000000$$

```
> f[x6,x7]:=(y7-y6)/h;
```

$$f_{.2,\,.4} := -1.500000000$$

```
> f[x7,x8]:=(y8-y7)/h;
```

$$f_{.4,\,.6} := -.5000000000$$

```
> f[x8,x9]:=(y9-y8)/h;
```

$$f_{.6,\ .8} := -.2058823530$$

```
> f[x9,x10]:=(y10-y9)/h;
```

$$f_{.8,\ 1.0} := -.1018099548$$

```
>
```

TABELA DAS DIFERENÇAS DIVIDIDAS: ODEM 2

```
> f[x0,x1,x2]:=(f[x1,x2]-f[x0,x1])/(2*h);
```

$$f_{-1,\ -.8,\ -.6} := .2601809955$$

```
> f[x1,x2,x3]:=(f[x2,x3]-f[x1,x2])/(2*h);
```

$$f_{-.8,\ -.6,\ -.4} := .7352941175$$

```
> f[x2,x3,x4]:=(f[x3,x4]-f[x2,x3])/(2*h);
```

$$f_{-.6,\ -.4,\ -.2} := 2.500000000$$

```
> f[x3,x4,x5]:=(f[x4,x5]-f[x3,x4])/(2*h);
```

$$f_{-.4,\ -.2,\ 0} := 2.500000000$$

```
> f[x4,x5,x6]:=(f[x5,x6]-f[x4,x5])/(2*h);
```

$$f_{-.2,\ 0,\ .2} := -12.50000000$$

```
> f[x5,x6,x7]:=(f[x6,x7]-f[x5,x6])/(2*h);
```

$$f_{0,\ .2,\ .4} := 2.500000000$$

```
> f[x6,x7,x8]:=(f[x7,x8]-f[x6,x7])/(2*h);
```

$$f_{.2,\ .4,\ .6} := 2.500000000$$

```
> f[x7,x8,x9]:=(f[x8,x9]-f[x7,x8])/(2*h);
```

$$f_{.4,\ .6,\ .8} := .7352941175$$

```
> f[x8,x9,x10]:=(f[x9,x10]-f[x8,x9])/(2*h);
```

$$f_{.6,\ .8,\ 1.0} := .2601809955$$

```
>
```

TABELA DAS DIFERENÇAS DIVIDIDAS: ODEM 3

```
> f[x0,x1,x2,x3]:=(f[x1,x2,x3]-f[x0,x1,x2])/(3*h);
```

$$f_{-1,\ -.8,\ -.6,\ -.4} := .7918552033$$

```
> f[x1,x2,x3,x4]:=(f[x2,x3,x4]-f[x1,x2,x3])/(3*h);
```

$$f_{-.8,\ -.6,\ -.4,\ -.2} := 2.941176472$$

```
> f[x2,x3,x4,x5]:=(f[x3,x4,x5]-f[x2,x3,x4])/(3*h);
```

$$f_{-.6,\ -.4,\ -.2,\ 0} := 0$$

```
> f[x3,x4,x5,x6]:=(f[x4,x5,x6]-f[x3,x4,x5])/(3*h);
```

$$f_{-.4,\ -.2,\ 0,\ .2} := -25.00000000$$

```
> f[x4,x5,x6,x7]:=(f[x5,x6,x7]-f[x4,x5,x6])/(3*h);
```

$$f_{-.2,\ 0,\ .2,\ .4} := 25.00000000$$

```
> f[x5,x6,x7,x8]:=(f[x6,x7,x8]-f[x5,x6,x7])/(3*h);
```

$$f_{0,\ .2,\ .4,\ .6} := 0$$

```
> f[x6,x7,x8,x9]:=(f[x7,x8,x9]-f[x6,x7,x8])/(3*h);
```

$$f_{.2,\ .4,\ .6,\ .8} := -2.941176472$$

```
> f[x7,x8,x9,x10]:=(f[x8,x9,x10]-f[x7,x8,x9])/(3*h);
```

$$f_{.4,\ .6,\ .8,\ 1.0} := -.7918552033$$

TABELA DAS DIFERENÇAS DIVIDIDAS: ODEM 4

```
> f[x0,x1,x2,x3,x4]:=(f[x1,x2,x3,x4]-f[x0,x1,x2,x3])/(4*h);
```

$$f_{-1,\ -.8,\ -.6,\ -.4,\ -.2} := 2.686651588$$

```
> f[x1,x2,x3,x4,x5]:=(f[x2,x3,x4,x5]-f[x1,x2,x3,x4])/(4*h);
```

$$f_{-.8,\ -.6,\ -.4,\ -.2,\ 0} := -3.676470590$$

```
> f[x2,x3,x4,x5,x6]:=(f[x3,x4,x5,x6]-f[x2,x3,x4,x5])/(4*h);
```

$$f_{-.6,\ -.4,\ -.2,\ 0,\ .2} := -31.25000000$$

```
> f[x3,x4,x5,x6,x7]:=(f[x4,x5,x6,x7]-f[x3,x4,x5,x6])/(4*h);
```

$$f_{-.4,\ -.2,\ 0,\ .2,\ .4} := 62.50000000$$

```
> f[x4,x5,x6,x7,x8]:=(f[x5,x6,x7,x8]-f[x4,x5,x6,x7])/(4*h);
```

$$f_{-.2,\ 0,\ .2,\ .4,\ .6} := -31.25000000$$

```
> f[x5,x6,x7,x8,x9]:=(f[x6,x7,x8,x9]-f[x5,x6,x7,x8])/(4*h);
```

$$f_{0,\ .2,\ .4,\ .6,\ .8} := -3.676470590$$

```
> f[x6,x7,x8,x9,x10]:=(f[x7,x8,x9,x10]-f[x6,x7,x8,x9])/(4*h);
```

$$f_{.2,\ .4,\ .6,\ .8,\ 1.0} := 2.686651588$$

TABELA DAS DIFERENÇAS DIVIDIDAS: ODEM 5

```
> f[x0,x1,x2,x3,x4,x5]:=(f[x1,x2,x3,x4,x5]-f[x0,x1,x2,x3,x4])/(5*h);
```

$$f_{-1,\ -.8,\ -.6,\ -.4,\ -.2,\ 0} := -6.363122178$$

```
> f[x1,x2,x3,x4,x5,x6]:=(f[x2,x3,x4,x5,x6]-f[x1,x2,x3,x4,x5])/(5*h);
```

$$f_{-.8,\ -.6,\ -.4,\ -.2,\ 0,\ .2} := -27.57352942$$

```
> f[x2,x3,x4,x5,x6,x7]:=(f[x3,x4,x5,x6,x7]-f[x2,x3,x4,x5,x6])/(5*h);
```

$$f_{-.6,\ -.4,\ -.2,\ 0,\ .2,\ .4} := 93.75000000$$

```
> f[x3,x4,x5,x6,x7,x8]:=(f[x4,x5,x6,x7,x8]-f[x3,x4,x5,x6,x7])/(5*h);
```

$$f_{-.4,\ -.2,\ 0,\ .2,\ .4,\ .6} := -93.75000000$$

```
> f[x4,x5,x6,x7,x8,x9]:=(f[x5,x6,x7,x8,x9]-f[x4,x5,x6,x7,x8])/(5*h);
```

$$f_{-.2,\ 0,\ .2,\ .4,\ .6,\ .8} := 27.57352942$$

```
> f[x5,x6,x7,x8,x9,x10]:=(f[x6,x7,x8,x9,x10]-f[x5,x6,x7,x8,x9])/(5*h
  );
```

$$f_{0,\ .2,\ .4,\ .6,\ .8,\ 1.0} := 6.363122178$$

TABELA DAS DIFERENÇAS DIVIDIDAS: ODEM 6

```
> f[x0,x1,x2,x3,x4,x5,x6]:=(f[x1,x2,x3,x4,x5,x6]-f[x0,x1,x2,x3,x4,x5
  ])/(6*h);
```

$$f_{-1,\ -.8,\ -.6,\ -.4,\ -.2,\ 0,\ .2} := -17.67533937$$

```
> f[x1,x2,x3,x4,x5,x6,x7]:=(f[x2,x3,x4,x5,x6,x7]-f[x1,x2,x3,x4,x5,x6
  ])/(6*h);
```

$$f_{-.8,\,-.6,\,-.4,\,-.2,\,0,\,.2,\,.4} := 101.1029412$$

```
> f[x2,x3,x4,x5,x6,x7,x8]:=(f[x3,x4,x5,x6,x7,x8]-f[x2,x3,x4,x5,x6,x7
  ])/(6*h);
```

$$f_{-.6,\,-.4,\,-.2,\,0,\,.2,\,.4,\,.6} := -156.2500000$$

```
> f[x3,x4,x5,x6,x7,x8,x9]:=(f[x4,x5,x6,x7,x8,x9]-f[x3,x4,x5,x6,x7,x8
  ])/(6*h);
```

$$f_{-.4,\,-.2,\,0,\,.2,\,.4,\,.6,\,.8} := 101.1029412$$

```
> f[x4,x5,x6,x7,x8,x9,x10]:=(f[x5,x6,x7,x8,x9,x10]-f[x4,x5,x6,x7,x8,
  x9])/(6*h);
```

$$f_{-.2,\,0,\,.2,\,.4,\,.6,\,.8,\,1.0} := -17.67533937$$

TABELA DAS DIFERENÇAS DIVIDIDAS: ODEM 7

```
> f[x0,x1,x2,x3,x4,x5,x6,x7]:=(f[x1,x2,x3,x4,x5,x6,x7]-f[x0,x1,x2,x3
  ,x4,x5,x6])/(7*h);
```

$$f_{-1,\,-.8,\,-.6,\,-.4,\,-.2,\,0,\,.2,\,.4} := 84.84162900$$

```
> f[x1,x2,x3,x4,x5,x6,x7,x8]:=(f[x2,x3,x4,x5,x6,x7,x8]-f[x1,x2,x3,x4
  ,x5,x6,x7])/(7*h);
```

$$f_{-.8,\,-.6,\,-.4,\,-.2,\,0,\,.2,\,.4,\,.6} := -183.8235294$$

```
> f[x2,x3,x4,x5,x6,x7,x8,x9]:=(f[x3,x4,x5,x6,x7,x8,x9]-f[x2,x3,x4,x5
  ,x6,x7,x8])/(7*h);
```

$$f_{-.6,\,-.4,\,-.2,\,0,\,.2,\,.4,\,.6,\,.8} := 183.8235294$$

```
> f[x3,x4,x5,x6,x7,x8,x9,x10]:=(f[x4,x5,x6,x7,x8,x9,x10]-f[x3,x4,x5,
  x6,x7,x8,x9])/(7*h);
```

$$f_{-.4,\,-.2,\,0,\,.2,\,.4,\,.6,\,.8,\,1.0} := -84.84162900$$

TABELA DAS DIFERENÇAS DIVIDIDAS: ODEM 8

```
> f[x0,x1,x2,x3,x4,x5,x6,x7,x8]:=(f[x1,x2,x3,x4,x5,x6,x7,x8]-f[x0,x1
  ,x2,x3,x4,x5,x6,x7])/(8*h);
```

$$f_{-1,\,-.8,\,-.6,\,-.4,\,-.2,\,0,\,.2,\,.4,\,.6} := -167.9157240$$

```
> f[x1,x2,x3,x4,x5,x6,x7,x8,x9]:=(f[x2,x3,x4,x5,x6,x7,x8,x9]-f[x1,x2
  ,x3,x4,x5,x6,x7,x8])/(8*h);
```

$$f_{-.8,\,-.6,\,-.4,\,-.2,\,0,\,.2,\,.4,\,.6,\,.8} := 229.7794118$$

```
> f[x2,x3,x4,x5,x6,x7,x8,x9,x10]:=(f[x3,x4,x5,x6,x7,x8,x9,x10]-f[x2,
  x3,x4,x5,x6,x7,x8,x9])/(8*h);
```

$$f_{-.6,\,-.4,\,-.2,\,0,\,.2,\,.4,\,.6,\,.8,\,1.0} := -167.9157240$$

TABELA DAS DIFERENÇAS DIVIDIDAS: ODEM 9

```
> f[x0,x1,x2,x3,x4,x5,x6,x7,x8,x9]:=(f[x1,x2,x3,x4,x5,x6,x7,x8,x9]-f
  [x0,x1,x2,x3,x4,x5,x6,x7,x8])/(9*h);
```

$$f_{-1,\,-.8,\,-.6,\,-.4,\,-.2,\,0,\,.2,\,.4,\,.6,\,.8} := 220.9417421$$

```
> f[x1,x2,x3,x4,x5,x6,x7,x8,x9,x10]:=(f[x2,x3,x4,x5,x6,x7,x8,x9,x10]
  -f[x1,x2,x3,x4,x5,x6,x7,x8,x9])/(9*h);
```

$$f_{-.8,\,-.6,\,-.4,\,-.2,\,0,\,.2,\,.4,\,.6,\,.8,\,1.0} := -220.9417421$$

TABELA DAS DIFERENÇAS DIVIDIDAS: ODEM 10

```
> f[x0,x1,x2,x3,x4,x5,x6,x7,x8,x9,x10]:=(f[x1,x2,x3,x4,x5,x6,x7,x8,x
  9,x10]-f[x0,x1,x2,x3,x4,x5,x6,x7,x8,x9])/(10*h);
```

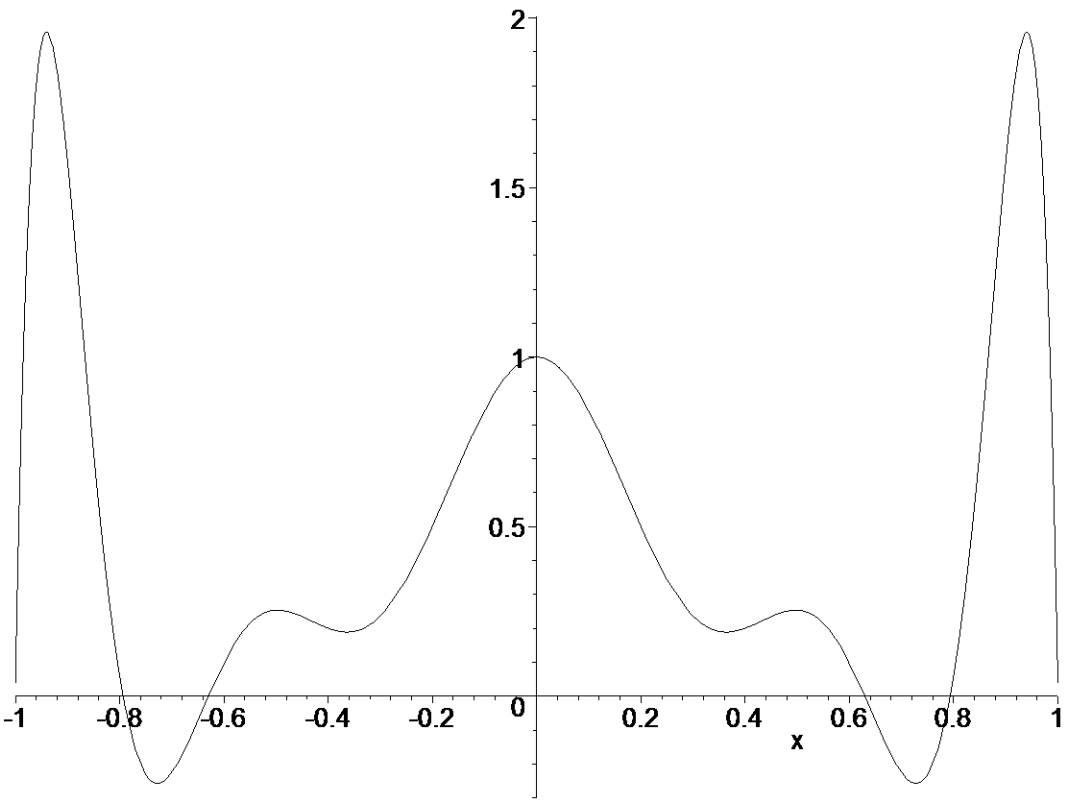
$$f_{-1,\,-.8,\,-.6,\,-.4,\,-.2,\,0,\,.2,\,.4,\,.6,\,.8,\,1.0} := -220.9417421$$

POLINOMIO INTERPOLADOR DE GRAU 10

```
> P10:=y0+f[x0,x1]*(x-x0)+f[x0,x1,x2]*(x-x0)*(x-x1)+f[x0,x1,x2,x3]*(
  x-x0)*(x-x1)*(x-x2)+f[x0,x1,x2,x3,x4]*(x-x0)*(x-x1)*(x-x2)*(x-x3)+
  f[x0,x1,x2,x3,x4,x5]*(x-x0)*(x-x1)*(x-x2)*(x-x3)*(x-x4)+f[x0,x1,x2
  ,x3,x4,x5,x6]*(x-x0)*(x-x1)*(x-x2)*(x-x3)*(x-x4)*(x-x5)+f[x0,x1,x2
  ,x3,x4,x5,x6,x7]*(x-x0)*(x-x1)*(x-x2)*(x-x3)*(x-x4)*(x-x5)*(x-x6)+
  f[x0,x1,x2,x3,x4,x5,x6,x7,x8]*(x-x0)*(x-x1)*(x-x2)*(x-x3)*(x-x4)*(
  x-x5)*(x-x6)*(x-x7)+f[x0,x1,x2,x3,x4,x5,x6,x7,x8,x9]*(x-x0)*(x-x1)
  *(x-x2)*(x-x3)*(x-x4)*(x-x5)*(x-x6)*(x-x7)*(x-x8)+f[x0,x1,x2,x3,x4
  ,x5,x6,x7,x8,x9,x10]*(x-x0)*(x-x1)*(x-x2)*(x-x3)*(x-x4)*(x-x5)*(x-
  x6)*(x-x7)*(x-x8)*(x-x9);
```

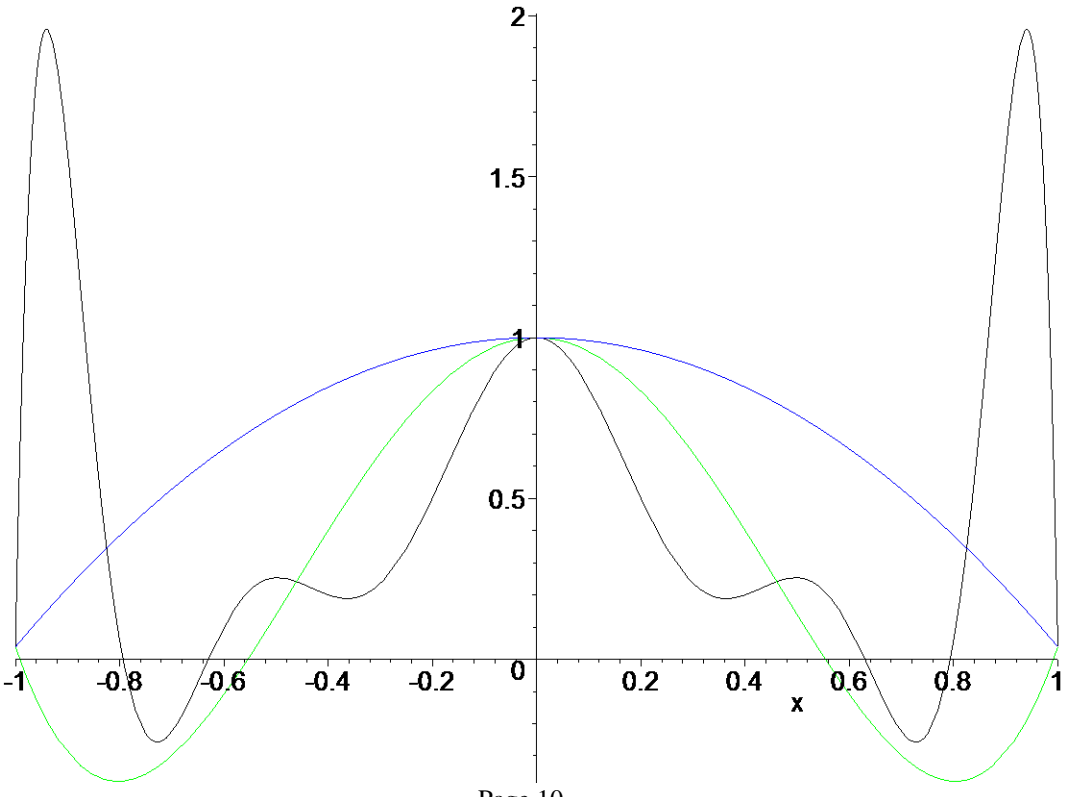
$$\begin{aligned} P10 :=\ & .1402714933 + .1018099548\,x + .2601809955\,(x+1)\,(x+.8) \\ & + .7918552033\,(x+1)\,(x+.8)\,(x+.6) + 2.686651588\,(x+1)\,(x+.8)\,(x+.6)\,(x+.4) \\ & - 6.363122178\,(x+1)\,(x+.8)\,(x+.6)\,(x+.4)\,(x+.2) \\ & - 17.67533937\,(x+1)\,(x+.8)\,(x+.6)\,(x+.4)\,(x+.2)\,x \\ & + 84.84162900\,(x+1)\,(x+.8)\,(x+.6)\,(x+.4)\,(x+.2)\,x\,(x-.2) \\ & - 167.9157240\,(x+1)\,(x+.8)\,(x+.6)\,(x+.4)\,(x+.2)\,x\,(x-.2)\,(x-.4) \\ & + 220.9417421\,(x+1)\,(x+.8)\,(x+.6)\,(x+.4)\,(x+.2)\,x\,(x-.2)\,(x-.4)\,(x-.6) \\ & - 220.9417421\,(x+1)\,(x+.8)\,(x+.6)\,(x+.4)\,(x+.2)\,x\,(x-.2)\,(x-.4)\,(x-.6)\,(x-.8) \end{aligned}$$

```
> Runge10:=plot([P10(x)], x=-1.0..1.0, color=[blue], style=[line]):
> plot([P10(x)], x=-1.0..1.0, color=[black], style=[line]);
```



COMPARAÇÃO EENTRE OS GRAFICOS DA FUNÇÃO E OS POLINOMIOS DE GRAU 2, 4 E 10

```
> plot([f(x), P2(x),P4(x), P10(x)], x=-1.0..1.0,
  color=[red,blue,green,black], style=[point,line,line,line]);
```



```
>
>
>
>
```

SPLINE CÚBICO

```
> f:=x->(1/(1+25*x^2)):
>
> x0:=-1:
> x1:=-0.8:
> x2:=-0.6:
> x3:=-0.4:
> x4:=-0.2:
> x5:=0.0:
> x6:=0.2:
> x7:=0.4:
> x8:=0.6:
> x9:=0.8:
> x10:=1.0:
```

VALOR DA FUNÇÃO

```
> y0:=f(x0):
> y1:=f(x1):
> y2:=f(x2):
> y3:=f(x3):
> y4:=f(x4):
> y5:=f(x5):
> y6:=f(x6):
> y7:=f(x7):
> y8:=f(x8):
> y9:=f(x9):
> y10:=f(x10):
> h:=0.2:
>
```

$$h3 := 4.0$$

```
> X[0]:= array(1..11,[x0,x1,x2,x3,x4,x5,x6,x7,x8,x9,x10]):;
> b1:=6*((y2-y1)/h-(y1-y0)/h);
```

$$b1 := .6244343892$$

```
> b2:=6*((y3-y2)/h-(y2-y1)/h);
```

$$b2 := 1.764705882$$

```
> b3:=6*((y4-y3)/h-(y3-y2)/h);
```

$$b3 := 6.000000000$$

```
> b4:=6*((y5-y4)/h-(y4-y3)/h);
```

$$b4 := 6.000000000$$

```
> b5:=6*((y6-y5)/h-(y5-y4)/h);
```

$$b5 := -30.00000000$$

```
> b6:=6*((y7-y6)/h-(y6-y5)/h);
```

$$b6 := 6.000000000$$

```
> b7:=6*((y8-y7)/h-(y7-y6)/h);
```

$$b7 := 6.000000000$$

```
> b8:=6*((y9-y8)/h-(y8-y7)/h);
```

$$b8 := 1.764705882$$

```
> b9:=6*((y10-y9)/h-(y9-y8)/h);
```

$$b9 := .6244343892$$

```
>
>
> a11:=2*(h+h):;
> a12:=(h):;
> a21:=a12:
> a22:=2*(h+h):;
> A:=matrix(9,9,[[a11,a12,0,0,0,0,0,0,0],[a21,a22,a21,0,0,0,0,0,0],[
  0,a21,a22,a21,0,0,0,0,0],[0,0,a21,a22,a21,0,0,0,0],[0,0,0,a21,a22,
  a21,0,0,0],[0,0,0,0,a21,a22,a21,0,0],[0,0,0,0,0,a21,a22,a21,0],[0,
  0,0,0,0,0,a21,a22,a21],[0,0,0,0,0,0,0,a21,a22]]);
```

$$A := \begin{bmatrix} .8 & .2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ .2 & .8 & .2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .2 & .8 & .2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .2 & .8 & .2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .2 & .8 & .2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .2 & .8 & .2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .2 & .8 & .2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .2 & .8 & .2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & .2 & .8 \end{bmatrix}$$

```
> b:=vector(9,[b1,b2,b3,b4,b5,b6,b7,b8,b9]);
```

$$b := [\,.6244343892,\ 1.764705882,\ 6.000000000,\ 6.000000000,\ -30.00000000,\ 6.000000000,\ 6.000000000,\ 1.764705882,\ .6244343892\,]$$

```
>
> Y2:=linsolve(A,b);
```

$$Y2 := [\,.4100522490,\ 1.481962950,\ 2.485625360,\ 18.57553561,\ -46.78776780,\ 18.57553561,\ 2.485625360,\ 1.481962950,\ .4100522490\,]$$

```
>
> Y_2:=
  vector(11,[0,Y2[1],Y2[2],Y2[3],Y2[4],Y2[5],Y2[6],Y2[7],Y2[8],Y2[9]
  ,0]);
```

$$Y\_2 := [\,0,\ .4100522490,\ 1.481962950,\ 2.485625360,\ 18.57553561,\ -46.78776780,\ 18.57553561,$$

2.485625360, 1.481962950, .4100522490, 0]

VETOR DAS SEGUNDA DERIVADA

```
> W:=vector(11,[0,Y2[1],Y2[2],Y2[3],Y2[4],Y2[5],Y2[6],Y2[7],Y2[8],Y2
  [9],0]);
```

W := [0, .4100522490, 1.481962950, 2.485625360, 18.57553561, -46.78776780, 18.57553561, 2.485625360, 1.481962950, .4100522490, 0]

```
>
>
```

PRIMEIRA DERIVADA

```
> z0:=(y1-y0)/h-h/6*(Y_2[2]+2*Y_2[1]);
```

$z0 := .08814154650$

```
> z1:=(y2-y1)/h-h/6*(Y_2[3]+2*Y_2[2]);
```

$z1 := .1291467714$

```
> z2:=(y3-y2)/h-h/6*(Y_2[4]+2*Y_2[3]);
```

$z2 := .3183482913$

```
> z3:=(y4-y3)/h-h/6*(Y_2[5]+2*Y_2[4]);
```

$z3 := .7151071223$

```
> z4:=(y5-y4)/h-h/6*(Y_2[6]+2*Y_2[5]);
```

$z4 := 2.821223219$

```
> z5:=(y6-y5)/h-h/6*(Y_2[7]+2*Y_2[6]);
```

$z5 := 0$

```
> z6:=(y7-y6)/h-h/6*(Y_2[8]+2*Y_2[7]);
```

$z6 := -2.821223219$

```
> z7:=(y8-y7)/h-h/6*(Y_2[9]+2*Y_2[8]);
```

$z7 := -.7151071223$

```
> z8:=(y9-y8)/h-h/6*(Y_2[10]+2*Y_2[9]);
```

$z8 := -.3183482913$

```
> z9:=(y10-y9)/h-h/6*(Y_2[11]+2*Y_2[10]);
```

$z9 := -.1291467714$

VETOR DA PRIMEIRA DERIVADA

```
> Z:= vector(10,[z0,z1,z2,z3,z4,z5,z6,z7,z8,z9]);
```

Z := [.08814154650, .1291467714, .3183482913, .7151071223, 2.821223219, 0, -2.821223219, -.7151071223, -.3183482913, -.1291467714]

CALCULO DO SPLINE CÚBICO

```
>
> C1:=y0+z0*(x-x0)+W[1]/2*(x-x0)^2+1/(6*h)*(W[2]-W[1])*(x-x0)^3;
```

$$C1 := .1266030850 + .08814154650\,x + .3417102075\,(x+1)^3$$

```
> C2:=y1+z1*(x-x1)+W[2]/2*(x-x1)^2+1/(6*h)*(W[3]-W[2])*(x-x1)^3;
```

$$C2 := .1621409465 + .1291467714\,x + .2050261245\,(x+.8)^2 + .8932589175\,(x+.8)^3$$

```
> C3:=y2+z2*(x-x2)+W[3]/2*(x-x2)^2+1/(6*h)*(W[4]-W[3])*(x-x2)^3;
```

$$C3 := .2910089748 + .3183482913\,x + .7409814750\,(x+.6)^2 + .8363853417\,(x+.6)^3$$

```
> C4:=y3+z3*(x-x3)+W[4]/2*(x-x3)^2+1/(6*h)*(W[5]-W[4])*(x-x3)^3;
```

$$C4 := .4860428489 + .7151071223\,x + 1.242812680\,(x+.4)^2 + 13.40825854\,(x+.4)^3$$

```
> C5:=y4+z4*(x-x4)+W[5]/2*(x-x4)^2+1/(6*h)*(W[6]-W[5])*(x-x4)^3;
```

$$C5 := 1.064244644 + 2.821223219\,x + 9.287767805\,(x+.2)^2 - 54.46941952\,(x+.2)^3$$

```
> C6:=y5+z5*(x-x5)+W[6]/2*(x-x5)^2+1/(6*h)*(W[7]-W[6])*(x-x5)^3;
```

$$C6 := 1 - 23.39388390\,x^2 + 54.46941952\,x^3$$

```
> C7:=y6+z6*(x-x6)+W[7]/2*(x-x6)^2+1/(6*h)*(W[8]-W[7])*(x-x6)^3;
```

$$C7 := 1.064244644 - 2.821223219\,x + 9.287767805\,(x-.2)^2 - 13.40825854\,(x-.2)^3$$

```
> C8:=y7+z7*(x-x7)+W[8]/2*(x-x7)^2+1/(6*h)*(W[9]-W[8])*(x-x7)^3;
```

$$C8 := .4860428489 - .7151071223\,x + 1.242812680\,(x-.4)^2 - .8363853417\,(x-.4)^3$$

```
> C9:=y8+z8*(x-x8)+W[9]/2*(x-x8)^2+1/(6*h)*(W[10]-W[9])*(x-x8)^3;
```

$$C9 := .2910089748 - .3183482913\,x + .7409814750\,(x-.6)^2 - .8932589175\,(x-.6)^3$$

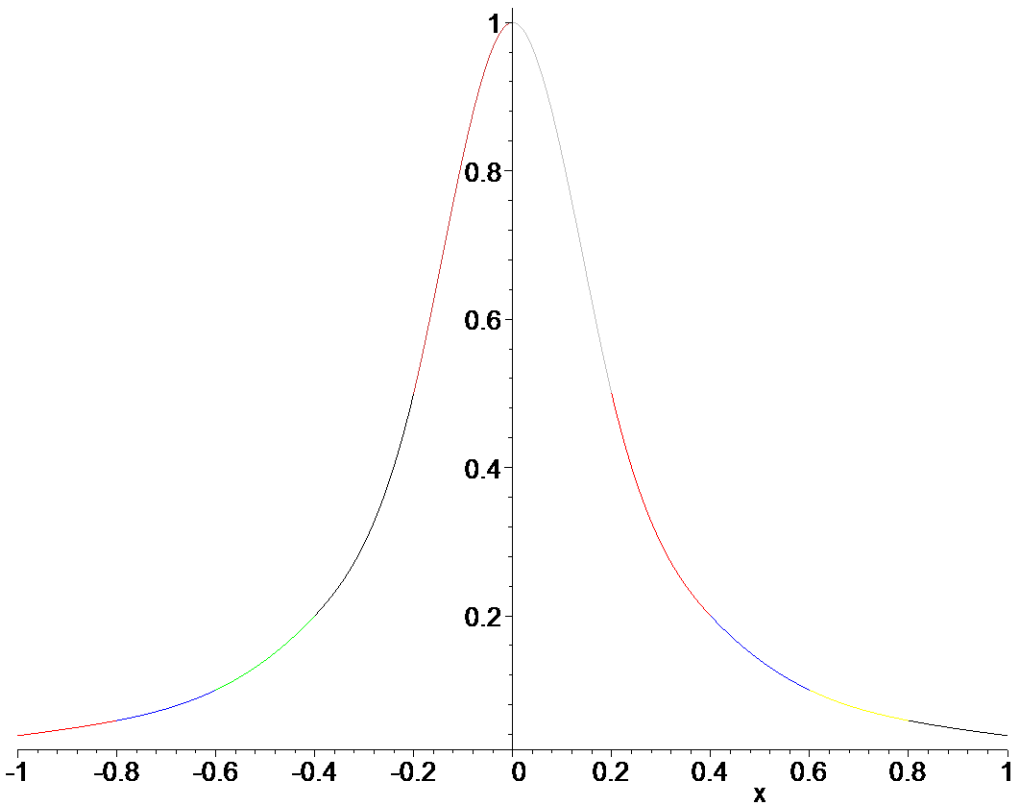
```
> C10:=y9+z9*(x-x9)+W[10]/2*(x-x9)^2+1/(6*h)*(W[11]-W[10])*(x-x9)^3;
```

$$C10 := .1621409465 - .1291467714\,x + .2050261245\,(x-.8)^2 - .3417102075\,(x-.8)^3$$

```
>
```

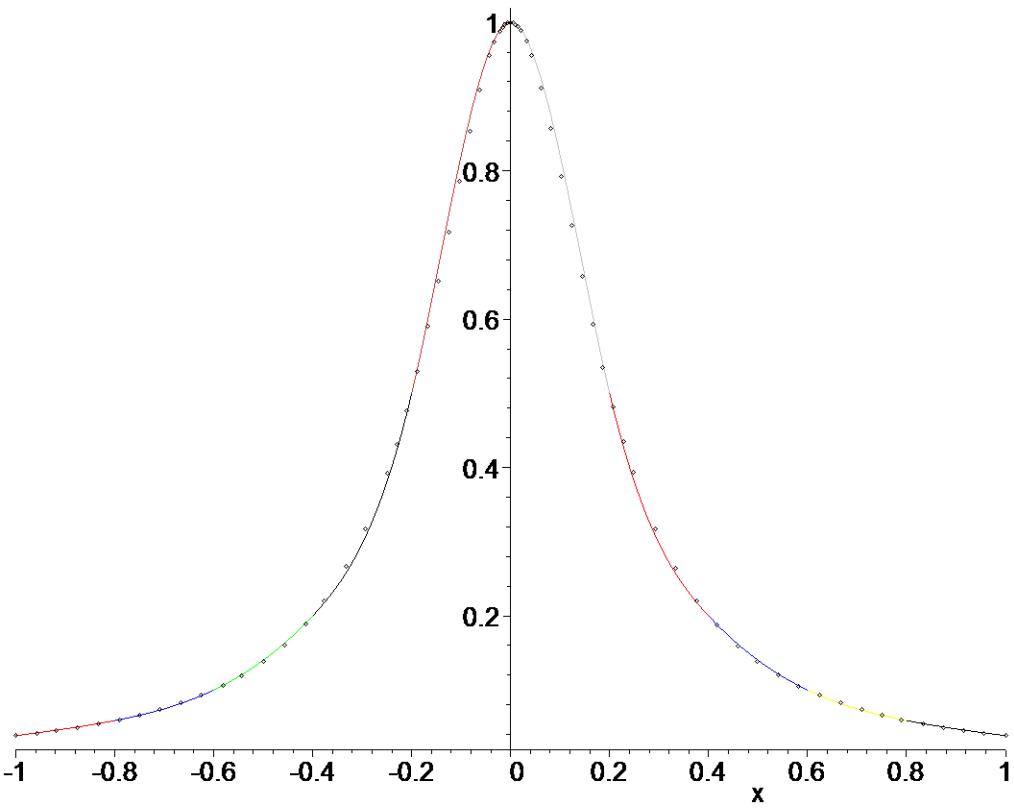
GRAFICO DA SPLINE CÚBICA

```
>
> sr1:=plot(C1, x=x0..x1, color=[red]):
> sr2:=plot(C2, x=x1..x2, color=[blue]):
> sr3:=plot(C3, x=x2..x3, color=[green]):
> sr4:=plot(C4, x=x3..x4, color=[black]):
> sr5:=plot(C5, x=x4..x5, color=[orange]):
> sr6:=plot(C6, x=x5..x6, color=[gray]):
> sr7:=plot(C7, x=x6..x7, color=[red]):
> sr8:=plot(C8, x=x7..x8, color=[blue]):
> sr9:=plot(C9, x=x8..x9, color=[yellow]):
> sr10:=plot(C10, x=x9..x10, color=[black]):
> display(sr1,sr2,sr3,sr4,sr5,sr6,sr7,sr8,sr9,sr10);
```



COMPARANDO O GRAFICO DA SPLINE CÚBICA COM A O GRAFICO DA FUNÇÃO INTERPOLADA f(X)

```
> z:=plot([f(x)], x=x0..x10, color=[black], style=[point]):
> display(sr1,sr2,sr3,sr4,sr5,sr6,sr7,sr8,sr9,sr10,z);
```


1
0.8
0.6
0.4
0.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
x

>