METODOS NUMÉRICOS PARA O CALCULO DE INTEGRAL DE MÚLTIPLAS VARIÁVEIS

```
> restart:
> with(plots):
> with(linalg):
```

I

EXEMPLO 1

INTEGRAL DUPLA

MÉTODO DOS TRAPÉZIOS a=x0<x1<<<<<xn=b , e c=y0<y1<<<<<<<<ym=d :

```
> INT:=(x,y)->int(f(x,y), x=1..2,y=0..1);
```

$$(x, y) \rightarrow int(f(x, y), x = 1..2, y = 0..1) \quad \textbf{(1)}$$

> ?

> $f := (x, y) \rightarrow (\exp(x + y));$

$$(x, y) \rightarrow e^{x+y} \quad \textbf{(2)}$$

> ?

> $exato := (\exp(2) - \exp(1)) \cdot (\exp(1) - \exp(0));$ # *VALOR EXATO DA INTEGRAL*

$$\left(e^2 - e\right) (e - 1) \quad \textbf{(3)}$$

> $evalf(\%);$

$$8.025706553 \quad \textbf{(4)}$$

MÉTODO DOS TRAPEZIOS

$$`?` \quad \textbf{(5)}$$

```
> g:=x->int(f(x,y), y=c..d);
```

$$x \rightarrow \int_c^d f(x, y)\, dy \quad \textbf{(6)}$$

Ith=It(h) é o método dos trapézios com base h

```
> n:=2:;
```

> $h := (x[n]-x[0])/n;$

$$\frac{1}{2} x_2 - \frac{1}{2} x_0 \quad \textbf{(7)}$$

```
> #Discretização do eixo x
> x[0]:=1.0:;
> x[n]:=2.0:;
> Itx:=0:;
> Itx:= ((f(x[0],y))+f(x[m],y));
```

(8)

$$e^{1.0+y} + e^{\frac{x}{m}+y} \quad \textbf{(8)}$$

```
> for j from 1 to n-1 do
> x[j]:=x[j-1]+h;
> Itx:= Itx+2*f(x[j],y);
> od:;
> Itx:=h/2*Itx;
```

$$0.2500000000\, e^{1.0+y} + 0.2500000000\, e^{\frac{x}{m}+y} + 0.5000000000\, e^{1.500000000+y} \quad \textbf{(9)}$$

```
> g:=y->.2500000000*exp(1.0+y)+.2500000000*exp(2.0+y)+.5000000000*
  exp(1.500000000+y);
```

$$y \rightarrow 0.2500000000\, e^{1.0+y} + 0.2500000000\, e^{2.0+y} + 0.5000000000\, e^{1.500000000+y} \quad \textbf{(10)}$$

> #g :=Itx;

`?` **(11)**

> $m := 2$:;

```
> y[0]:=0.0:;
> y[m]:=1.0:;
```

`?` **(12)**

```
> k:=(y[m]-y[0])/m; #Discretização do eixo y
```

0.5000000000 **(13)**

```
> Ity:=0:;
> Ity:= (g(y[0])+g(y[m]));
```

17.72757426 **(14)**

```
> for i from 1 to m-1 do
> y[i]:=y[i-1]+k;
> Ity:= Ity+2*g(y[i]):;
> od:;
> Ity:=k/2*Ity;
```

8.362180470 **(15)**

> $Ithk2 := Ity$;

8.362180470 **(16)**

```
> evalf(%);
```

8.362180470 **(17)**

> $Erro2 := evalf(\mathrm{abs}(Ity - exato))$;

0.336473917 **(18)**

MÉTODO DOS TRAPEZIOS -aumento a discretização no eixos para n=4

`?` **(19)**

```
> g:=x->int(f(x,y), y=c..d);
```

$$x \to \int_c^d f(x, y)\, dy \quad (20)$$

Ith=It(h) é o método dos trapézios com base h

```
> n:=4:;
> x[0]:=1.0:;
> x[n]:=2.0:;
```

$$`?` \quad (21)$$

```
> h := (x[n]-x[0])/n;
```

$$0.2500000000 \quad (22)$$

```
> #Discretização do eixo x
```

```
> Itx:=0:;
> Itx:= ((f(x[0],y))+f(x[n],y));
```

$$e^{1.0+y} + e^{2.0+y} \quad (23)$$

```
> for j from 1 to n-1 do
> x[j]:=x[j-1]+h;
> Itx:= Itx+2*f(x[j],y);
> od:;
> Itx:=h/2*Itx;
```

$$0.1250000000\, e^{1.0+y} + 0.1250000000\, e^{2.0+y} + 0.2500000000\, e^{1.250000000+y} + 0.2500000000\, e^{1.500000000+y} + 0.2500000000\, e^{1.750000000+y} \quad (24)$$

```
> evalf(%);
```

$$0.1250000000\, e^{1.0+y} + 0.1250000000\, e^{2.0+y} + 0.2500000000\, e^{1.250000000+y} + 0.2500000000\, e^{1.500000000+y} + 0.2500000000\, e^{1.750000000+y} \quad (25)$$

```
> g:=y->(Itx(y), y=c..d);
```

$$y \to (Itx(y), y = c..d) \quad (26)$$

```
> g(y) := 0.1250000000 e^(1.0+y) + 0.1250000000 e^(2.0+y) + 0.2500000000 e^(1.250000000+y) + 0.2500000000 e^(1.500000000+y) + 0.2500000000 e^(1.750000000+y)
```

$$y \to 0.1250000000\, e^{1.0+y} + 0.1250000000\, e^{2.0+y} + 0.2500000000\, e^{1.250000000+y} + 0.2500000000\, e^{1.500000000+y} + 0.2500000000\, e^{1.750000000+y} \quad (27)$$

```
> m := 4:;
> y[0]:=0.0:;
> y[m]:=1.0:;
```

$$`?` \quad (28)$$

```
> k:=(y[m]-y[0])/m; #Discretização do eixo y
```

$$0.2500000000 \quad (29)$$

```
> Ity:=0:;
> Ity:= (g(y[0])+g(y[m]));
```
17.45761547 **(30)**

```
> for i from 1 to m-1 do
> y[i]:=y[i-1]+k;
> Ity:= Ity+2*g(y[i]):;
> od:;
> Ity:=k/2*Ity;
```
8.109437970 **(31)**

$> Ithk4 := Ity;$

8.109437970 **(32)**

```
> Erro4 := evalf(abs(Ity-exato));
```
0.083731417 **(33)**

\`?\` **(34)**

MÉTODO DOS TRAPEZIOS -aumento a discretização no eixos para n=8

\`?\` **(35)**

```
> g:=x->int(f(x,y), y=c..d);
```
$$x \to \int_c^d f(x, y)\, dy \quad (36)$$

Ith=It(h) é o método dos trapézios com base h

```
> n:=8:;
> x[0]:=1.0:;
> x[n]:=2.0:;
```
\`?\` **(37)**

$> h := (x[n]-x[0])/n;$

0.1250000000 **(38)**

```
> #Discretização do eixo x

> Itx:=0:;
> Itx:= ((f(x[0],y))+f(x[n],y));
```
$$e^{1.0+y} + e^{2.0+y} \quad (39)$$

```
> for j from 1 to n-1 do
> x[j]:=x[j-1]+h;
> Itx:= Itx+2*f(x[j],y);
> od:;
> Itx:=h/2*Itx;
```
$$0.06250000000\, e^{1.0+y} + 0.06250000000\, e^{2.0+y} + 0.1250000000\, e^{1.125000000+y} \quad (40)$$

$+ 0.1250000000\, e^{1.250000000 + y} + 0.1250000000\, e^{1.375000000 + y}$
$+ 0.1250000000\, e^{1.500000000 + y} + 0.1250000000\, e^{1.625000000 + y}$
$+ 0.1250000000\, e^{1.750000000 + y} + 0.1250000000\, e^{1.875000000 + y}$

> *evalf*(%);

$$0.06250000000\, e^{1.0 + y} + 0.06250000000\, e^{2.0 + y} + 0.1250000000\, e^{1.125000000 + y} + 0.1250000000\, e^{1.250000000 + y} + 0.1250000000\, e^{1.375000000 + y} + 0.1250000000\, e^{1.500000000 + y} + 0.1250000000\, e^{1.625000000 + y} + 0.1250000000\, e^{1.750000000 + y} + 0.1250000000\, e^{1.875000000 + y} \quad \textbf{(41)}$$

```
> g:=y->(Itx(y), y=c..d);
```

$$y \rightarrow (Itx(y), y = c..d) \quad \textbf{(42)}$$

> $g(y) := 0.06250000000\, e^{1.0 + y} + 0.06250000000\, e^{2.0 + y} + 0.1250000000\, e^{1.125000000 + y} + 0.1250000000\, e^{1.250000000 + y} + 0.1250000000\, e^{1.375000000 + y} + 0.1250000000\, e^{1.500000000 + y} + 0.1250000000\, e^{1.625000000 + y} + 0.1250000000\, e^{1.750000000 + y} + 0.1250000000\, e^{1.875000000 + y}$:

> $m := 8$:;

```
> y[0]:=0.0:;
> y[m]:=1.0:;
```

$$`?` \quad \textbf{(43)}$$

```
> k:=(y[m]-y[0])/m; #Discretização do eixo y
```

$$0.1250000000 \quad \textbf{(44)}$$

```
> Ity:=0:;
> Ity:= (g(y[0])+g(y[m]));
```

$$17.38986282 \quad \textbf{(45)}$$

```
> for i from 1 to m-1 do
> y[i]:=y[i-1]+k;
> Ity:= Ity+2*g(y[i]):;
> od:;
> Ity:=k/2*Ity;
```

$$8.046614995 \quad \textbf{(46)}$$

> *Ithk8* := *Ity*;

$$8.046614995 \quad \textbf{(47)}$$

> *Erro8* := *evalf*(abs(*Ity* − *exato*));

$$0.020908442 \quad \textbf{(48)}$$

> *Erro2*; *Erro4*; *Erro8*; # *ERRO DO MÉTODO DOS TRAPÉZIOS*

$$0.336473917 \quad \textbf{(49)}$$
$$0.083731417$$
$$0.020908442$$

Note que o método dos trapézios o erro vai dimunindo quando aumentamos a discretização de m=n=2, m=n=4 e m=n=8
Erro8<Erro4<Erro2

MÉTODO DE ROMBERG PARA INTEGRAL DUPLA

> $R1hk2 := \frac{4 \cdot Ithk4 - Ithk2}{3}$;
ROMBERG DE PRIMEIRA ORDEM

8.025190470 **(50)**

> $Erro22 := evalf(\text{abs}(R1hk2 - exato))$;

0.000516083 **(51)**

> $R1hk4 := \frac{4 \cdot Ithk8 - Ithk4}{3}$;
ROMBERG DE PRIMEIRA ORDEM

8.025674000 **(52)**

> $Erro23 := evalf(\text{abs}(R1hk4 - exato))$;

0.000032553 **(53)**

> $R2hk2 := \frac{16 \cdot R1hk4 - R1hk2}{15}$;
ROMBERG DE SEGUNDA ORDEM

8.025706236 **(54)**

> $Erro22 := evalf(\text{abs}(R2hk2 - exato))$;

$3.17\ 10^{-7}$ **(55)**

\`?\` **(56)**

GAUSSIANA PARA CÁLCULO DA INTEGRAL DUPLA

GAUSSIANA COM UM PONTO NO INTERIOR DO ESPAÇO X E Y

```
> x[0]:=1.0:;
> x[n]:=2.0:;
```

```
> f := (x, y)-> exp(x+y);
```

$(x, y) \rightarrow e^{x+y}$ **(57)**

> ?
> ?

```
> g:=s->((x[n]-x[0])*s +(x[n]+x[0]))/2:;
> dx:=(x[n]-x[0])*1/2;
```

0.5000000000 **(58)**

```
> s1:=g(0);
```

1.500000000 **(59)**

> $evalf(\%)$;

1.500000000 **(60)**

> $Itx := (f(s1, y))$;

$e^{1.500000000 + y}$ **(61)**

```
> Gauss0:=dx*2*(f(s1,y)):;
> evalf(%);
```

(62)

$$1.000000000\, e^{1.500000000 + y} \quad (62)$$

```
> ?
```

$> U(y) := 1.000000000\, e^{1.500000000 + y};$

$$y \rightarrow 1.000000000\, e^{1.500000000 + y} \quad (63)$$

```
> y[0]:=0.0:;
> y[n]:=1.0:;
> ?
> v:=t->((y[n]-y[0])*t +(y[n]+y[0]))/2:;
> dy:=(y[n]-y[0])*1/2:;
> t1:=v(0);
```

$$0.5000000000 \quad (64)$$

```
> evalf(%);
```

$$0.5000000000 \quad (65)$$

```
> Gauss1:=dy*2*(U(t1)):;
> evalf(%);
```

$$7.389056100 \quad (66)$$

```
> ?
> ErroG1:= evalf(abs(Gauss1-exato));
```

$$0.636650453 \quad (67)$$

GAUSSIANA COM DOIS PONTOS NO INTERIOR DO ESPAÇO X E Y

```
> ?
> x[0]:=1.0:;
> x[n]:=2.0:;
```

```
> f := (x, y)-> exp(x+y);
```

$$(x, y) \rightarrow e^{x+y} \quad (68)$$

```
> ?
> g:=s->((x[n]-x[0])*s +(x[n]+x[0]))/2:;
> dx:=(x[n]-x[0])*1/2:;
> s1:=g(-sqrt(3)/3);
```

$$-0.1666666666\sqrt{3} + 1.500000000 \quad (69)$$

```
> evalf(%);
```

$$1.211324865 \quad (70)$$

```
> s2:=g(sqrt(3)/3);
```

$$0.1666666666\sqrt{3} + 1.500000000 \quad (71)$$

```
> evalf(%);
```

(72)

$$1.788675135 \quad (72)$$

> $Itx := ((f(s1, y)) + f(s2, y));$

$$e^{-0.1666666666\sqrt{3} + 1.500000000 + y} + e^{0.1666666666\sqrt{3} + 1.500000000 + y} \quad (73)$$

```
> Gauss2:=dx*(f(s1,y)+f(s2,y)):;
> evalf(%);
```

$$0.5000000000\, e^{1.211324865 + y} + 0.5000000000\, e^{1.788675135 + y} \quad (74)$$

> ?

> $U(y) := 0.5000000000\, e^{1.211324865 + y} + 0.5000000000\, e^{1.788675135 + y};$

$$y \rightarrow 0.5000000000\, e^{1.211324865 + y} + 0.5000000000\, e^{1.788675135 + y} \quad (75)$$

```
> y[0]:=0.0:;
> y[n]:=1.0:;
```

> ?

```
> v:=t->((y[n]-y[0])*t +(y[n]+y[0]))/2:;
> dy:=(y[n]-y[0])*1/2:;
> t1:=v(-sqrt(3)/3);
```

$$-0.1666666666\sqrt{3} + 0.5000000000 \quad (76)$$

> $evalf(\%);$

$$0.2113248654 \quad (77)$$

```
> t2:=v(sqrt(3)/3);
```

$$0.1666666666\sqrt{3} + 0.5000000000 \quad (78)$$

> $evalf(\%);$

$$0.7886751346 \quad (79)$$

```
> Gauss3:=dy*(U(t1)+U(t2)):;
> evalf(%);
```

$$8.022106255 \quad (80)$$

> ?

```
> ErroG2:= evalf(abs(Gauss3-exato));
```

$$0.003600298 \quad (81)$$

```
> evalf(%);
```

$$0.003600298 \quad (82)$$

GAUSSIANA COM TRÊS PONTOS NO INTERIOR DO ESPAÇO X E Y

> ?

```
> x[0]:=1.0:;
> x[n]:=2.0:;
```

```
> f := (x, y)-> exp(x+y);
```

$$(x, y) \rightarrow e^{x+y} \quad (83)$$

> ?

```
> g:=s->((x[n]-x[0])*s +(x[n]+x[0]))/2:;
> dx:=(x[n]-x[0])*1/2:;
> s1:=g(-sqrt(3/5));
```

$$-0.1000000000\sqrt{15} + 1.500000000 \quad (84)$$

> $evalf(\%);$

$$1.112701665 \quad (85)$$

> $s2 := g(0);$

$$1.500000000 \quad (86)$$

> $evalf(\%);$

$$1.500000000 \quad (87)$$

```
> s3:=g(sqrt(3/5));
```

$$0.1000000000\sqrt{15} + 1.500000000 \quad (88)$$

> $evalf(\%);$

$$1.887298335 \quad (89)$$

> $Itx := \left(\frac{5}{9} \cdot f(s1, y) + \frac{8}{9} \cdot f(s2, y) + \frac{5}{9} \cdot f(s3, y)\right);$

$$\frac{5}{9}\, e^{-0.1000000000\sqrt{15} + 1.500000000 + y} + \frac{8}{9}\, e^{1.500000000 + y} + \frac{5}{9}\, e^{0.1000000000\sqrt{15} + 1.500000000 + y} \quad (90)$$

```
> Gauss4:=dx*Itx;
```

$$0.2777777778\, e^{-0.1000000000\sqrt{15} + 1.500000000 + y} + 0.4444444444\, e^{1.500000000 + y} + 0.2777777778\, e^{0.1000000000\sqrt{15} + 1.500000000 + y} \quad (91)$$

```
> evalf(%);
```

$$0.2777777778\, e^{1.112701665 + y} + 0.4444444444\, e^{1.500000000 + y} + 0.2777777778\, e^{1.887298335 + y} \quad (92)$$

> ?

> $U(y) := 0.2777777778\, e^{-0.1000000000\sqrt{15} + 1.500000000 + y} + 0.4444444444\, e^{1.500000000 + y} + 0.2777777778\, e^{0.1000000000\sqrt{15} + 1.500000000 + y};$

$$y \rightarrow 0.2777777778\, e^{(-1) \cdot 0.1000000000\sqrt{15} + 1.500000000 + y} + 0.4444444444\, e^{1.500000000 + y} + 0.2777777778\, e^{0.1000000000\sqrt{15} + 1.500000000 + y} \quad (93)$$

```
> y[0]:=0.0:;
> y[n]:=1.0:;
```

> ?

```
> v:=t->((y[n]-y[0])*t +(y[n]+y[0]))/2:;
> dy:=(y[n]-y[0])*1/2:;
> t1:=v(-sqrt(3/5));
```

$$-0.1000000000\sqrt{15} + 0.5000000000 \quad (94)$$

> $evalf(\%);$

$$0.1127016654 \quad (95)$$

```
> t2:=v(0);
```

$$0.5000000000 \quad (96)$$

> *evalf*(%);

$$0.5000000000 \quad (97)$$

```
> t3:=v(sqrt(3/5));
```

$$0.1000000000\sqrt{15} + 0.5000000000 \quad (98)$$

> *evalf*(%);

$$0.8872983346 \quad (99)$$

> ?

> ?

```
> Gauss5:=dy*(5/9*U(t1)+8/9*U(t2)+5/9*U(t3));
```

$$0.07716049385\, e^{-0.2000000000\sqrt{15} + 2.000000000} + 0.2469135802\, e^{2.000000000 - 0.1000000000\sqrt{15}} + 2.599853072 + 0.2469135802\, e^{2.000000000 + 0.1000000000\sqrt{15}} + 0.07716049385\, e^{0.2000000000\sqrt{15} + 2.000000000} \quad (100)$$

> ?

> ?

```
> evalf(%);
```

$$8.025698855 \quad (101)$$

> ?

```
> ErroG3:= evalf(abs(Gauss5-exato));
```

$$0.000007698 \quad (102)$$

COMPRARAÇÃO ENTRE OS ERROS USANDO A GAUSSIANA COM UM PONTO ErroG1, dois pontos ErroG2 e três pontos ErroG3

> ?

> *ErroG1*; *ErroG2*; *ErroG3*

$$0.636650453 \quad (103)$$
$$0.003600298$$
$$0.000007698$$

> ?

> ?