FÓRMULAS

1. Método de Romberg:

   (a) $R_1(h) = (4I_t(h/2) - I_t(h))/3$ com precisão $O(h^4)$

   (b) $R_2(h) = (16R_1(h/2) - R_1(h))/15$, com precisão $O(h^6)$ onde $I_t(h)$ representa o método dos trapézios.

2. Quadratura Gaussiana: Transformação $x(\xi) = [(b-a)\xi + (b+a)]/2$.

   (a) Se $Nint = 1$ então $w_0 = 2$ e $x_0 = 0$

   Erro: $E_1 \leq \frac{1}{3} \max_{[-1,1]} |f''(x)|$.

   (b) Se $Nint = 2$ então $w_0 = w_1 = 1$; $-x_0 = x_1 = \sqrt{3}/3$

   Erro: $E_2 \leq \frac{1}{135} \max_{[-1,1]} |f^{iv}(x)|$

   (c) Se $Nint = 3$ então $w_0 = w_2 = 5/9, w_1 = 8/9$ associados aos pontos $x_0 = -\sqrt{\frac{3}{5}}, x_1 = 0, x_2 = \sqrt{\frac{3}{5}}$

   Erro: $E_2 \leq \frac{1}{15750} \max_{[-1,1]} |f^{vi}(x)|$

3. Método dos Mínimos Quadrados (Polinômios Ortogonais): Definindo $\varphi_1(x) = 1$; $\varphi_2(x) = x - \alpha_1$ onde $\alpha_1 = \frac{1}{m}\sum_{k=1}^{m} x_k$ e a sequência:

   $\varphi_{j+1}(x) = x\varphi_j(x) - \alpha_j\varphi_j(x) - \beta_{j-1}\varphi_{j-1}(x);\ j = 2, \cdots, n$ onde

$$\begin{cases} \alpha_j = \sum_{k=1}^{m} x_k\Big(\varphi_j(x_k)\Big)^2 \Big/ \sum_{k=1}^{m} \Big(\varphi_j(x_k)\Big)^2 & j = 2, \cdots, m-1 \\ \beta_{j-1} = \sum_{k=1}^{m} x_k\Big(\varphi_j(x_k)\varphi_{j-1}(x_k)\Big) \Big/ \sum_{k=1}^{m} \Big(\varphi_{j-1}(x_k)\Big)^2 & j = 2, \cdots, m-1 \end{cases}$$

   então obtemos o sistema linear $Ax = b$, com

$$a_{ij} = \sum_{k=1}^{m} \varphi_i(x_k)\varphi_j(x_k) \text{ e } \quad b_j = \sum_{k=1}^{m} f(x_k)\varphi_j(x_k)$$

4. SPLINE CÚBICO $\Rightarrow y''(x_0) = \alpha$ e $y''(x_n) = \beta$

$$\left| \begin{array}{l} 2(h_1 + h_2)y_1'' + h_2y_2'' = 6\left\{\frac{(y_2 - y_1)}{h_2} - \frac{(y_1 - y_0)}{h_1}\right\} - h_1\alpha \\ h_iy_{i-1}'' + 2(h_i + h_{i+1})y_i'' + h_{i+1}y_{i+1}'' = 6\left\{\frac{(y_{i+1} - y_i)}{h_{i+1}} - \frac{(y_i - y_{i-1})}{h_i}\right\}, \quad i = 2, ..n-2 \\ h_{n-1}y_{n-2}'' + 2(h_{n-1} + h_n)y_{n-1}'' = 6\left\{\frac{(y_n - y_{n-1})}{h_n} - \frac{(y_{n-1} - y_{n-2})}{h_{n-1}}\right\} - h_n\beta \end{array} \right.$$

$$y'_{i-1} = \frac{y_i - y_{i-1}}{h_i} - \frac{h_i}{6}(y''_i + 2y''_{i-1}), \quad i = 1, \cdots, n$$

$$C_i(x) = y_{i-1} + y'_{i-1}(x - x_{i-1}) + \frac{y''_{i-1}}{2}(x - x_{i-1})^2 + \frac{(y''_i - y''_{i-1})}{6h_i}(x - x_{i-1})^3,$$
$$i = 1, \cdots, n$$

Se $\alpha = \beta = 0$ então o SPLINE CÚBICO é chamado de NATURAL.

5. Algoritmo do Método dos Gradientes Conjugados

$$x_0 = b; \quad p_0 = r_0 = b - Ax_0;$$
$$Para \;\; k = 0, 1, \cdots$$
$$\alpha_k = (r_k, p_k)/(p_k, Ap_k); \qquad x_{k+1} = x_k + \alpha_k \, p_k$$
$$r_{k+1} = r_k - \alpha_k \, Ap_k. \quad Se \quad \|r_{k+1}\|_\infty = \max_{i=1\cdots,n} \{r^i_{k+1}\} \leq \varepsilon; \quad pare.$$

caso contrário; faça

$$\beta_k = (r_{k+1}, Ap_k)/(p_k, Ap_k); \qquad p_{k+1} = r_{k+1} - \beta_k p_k$$

Faça $\;k = k + 1,\;$ e volte para $\;\alpha_k$

6. Algoritmo de Cholesky:

$$l_{11} = (a_{11})^{1/2}$$
$$Para \; j = 2, \cdots, n; \qquad l_{j1} = a_{j1}/l_{11}$$
$$Para \; i = 2, \cdots, n - 1$$
$$l_{ii} = \left(a_{ii} - \sum_{k=1}^{i-1} l^2_{ik}\right)^{1/2}$$
$$Para \; j = i + 1, \cdots, n$$
$$l_{ji} = \frac{1}{l_{ii}} \left[a_{ji} - \sum_{k=1}^{i-1} l_{jk} l_{ik}\right]$$

$$l_{nn} = (a_{nn} - \sum_{k=1}^{n-1} l^2_{nk})^{1/2}$$